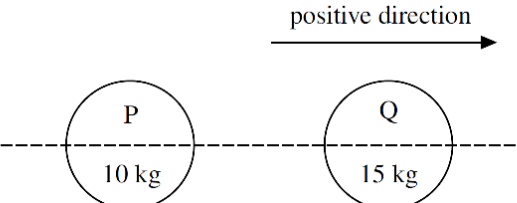| 1(a) i.	Taking PQ to be the positive direction $u_p = -1.75$ $F = 13$ $v_p = 4.75$ $t = T$	To find T we need to use SUVAT, however we need either a or s Using $F=ma$ $$13 = 10 \times a$$ $$\therefore a = 1.3 \text{ m/s}^2$$ Now use $v=u+at$ $$4.75 = -1.75 + 1.3T$$ $$\therefore 1.3T = 6.5$$ $$\therefore T = 5 \text{ seconds}$$
	For collisions it is important to remember • that directions are extremely important • annotate a diagram with your positive direction • anything going the opposite way has a negative velocity After collision, there can be several scenarios • they could coalesce • they could both go in positive direction • or both in negative direction • or separate in different directions • regardless of which outcome, particles cannot overtake one another	
ii.	Before collision $P = 10kg$ $u_p = 4.75$ $Q = 15kg$ $u_q = -0.5$ After collision then P & Q coalesce i.e. they become one particle $P + Q = 25kg$ $v = ?$	positive direction, P 10 kg, Q 15 kg Using conservation of momentum $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$ $$\therefore 10(4.75) + 15(-0.5) = 25(v)$$ $$\therefore 25v = 40$$ $$\therefore v = 1.6$$ This is positive and so is in direction PQ
iii.	Before collision $P = 10, u_p = 4.75$ $Q = 15, u_q = -0.5$ After collision particles separate $v_p = 1$ $v_q = ?$	Using conservation of momentum $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$ $$\therefore 10(4.75) + 15(-0.5) = 10(1) + 15(v_q)$$ $$\therefore 15v_q = 30$$ $$\therefore v_q = 2$$ This is positive and so is in direction PQ

The coefficient of restitution allows us to model when some of the energy dissipates through bounces, and is e.

- We use smooth planes for impact
- Restitution affects only the component of velocity which is perpendicular to the bounce surface, not the component parallel
- *Speed of separation = e x speed of approach*
- $e = \dfrac{v_2 - v_1}{u_1 - u_2} = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right)$ (two balls) or $v = e \times u$ (single ball)

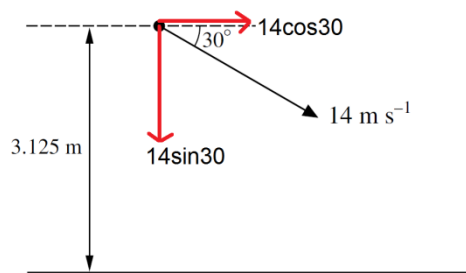|	
$u_p = 4.75$ $u_q = -0.5$ $v_p = 1$ $v_q = 2$	$e = \dfrac{v_2 - v_1}{u_1 - u_2} = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right)$ $\therefore e = \dfrac{2-1}{4.75 - (-0.5)} = \dfrac{1}{5.25} = 0.190(3sf)$

1(b)

Horizontally
$u_h = 14\cos 30$
$v_h = 14\cos 30$

Vertically
$u_v = 14\sin 30 = 7$
$v_v = ?$
$a = 9.8$
$s = 3.125$

The velocity as ball hits the ground is
$$v_v^2 = u_v^2 + 2as$$
$$v_v^2 = 7^2 + 2\times 9.8 \times 3.125$$
$$v_v = 10.5$$

After the first bounce
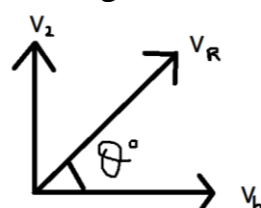$$v_1 = v_v \times e$$
$$v_1 = 10.5 \times 0.6$$

After the second bounce
$$v_2 = v_1 \times e$$
$$v_2 = 10.5 \times 0.6 \times 0.6$$

The resultant velocity is found from v_2 & v_h

The angle that the ball leaves 2$^{\text{nd}}$ bounce is

$$\tan\theta = \dfrac{v_2}{v_h} = \dfrac{10.5 \times 0.6^2}{14\cos 30}$$
$$\therefore \theta = 17.3°$$

2	• This question requires non-exact answers to be given to 4 sig fig.
	• Draw diagrams and annotate with arrows the directions of forces
	• Resolve into parallel and perpendicular components as necessary
	• Tell the examiner what methods you are using in your calcs
	• Directions are important – a diagram may suffice
	• Use the memory keys to store accurate answers, do not use rounded answers in further calculations

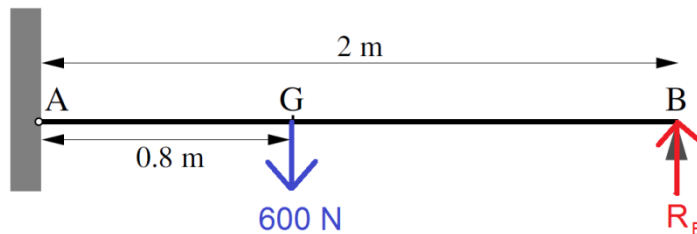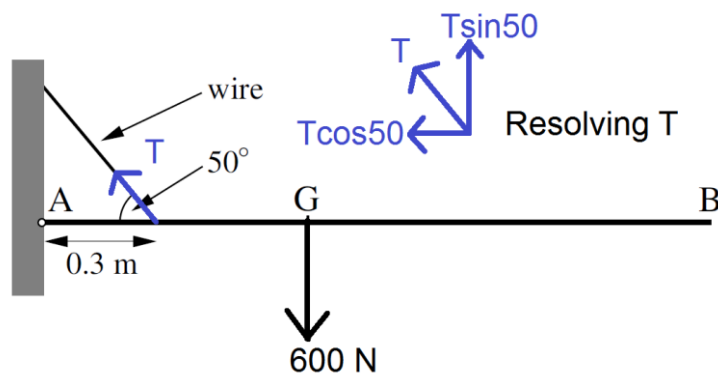| i.	W=600 N Support at B is R_B Upwards There are forces X and Y at A but not used here yet	 Taking moments about A (to exclude X and Y there) $$0.8 \times 600 = 2 \times R_B$$ $$\therefore R_B = 240N$$ As shown in the diagram this reaction force at B is upwards
ii.	Tension in wire must be resolved into horizontal and vertical components	 Taking moments about A $$0.3 \times T \sin 50 = 0.8 \times 600$$ $$\therefore T \sin 50 = 1600$$ $$\therefore T = \frac{1600}{\sin 50} = 2088.65.....$$ $$\therefore T = 2089N \quad \text{(4 sf)}$$

iii. To find X,Y,R and angle
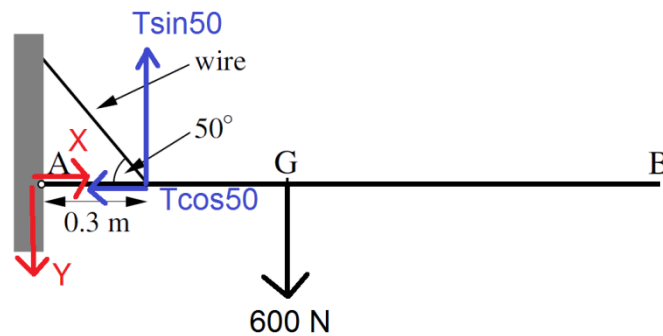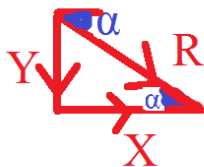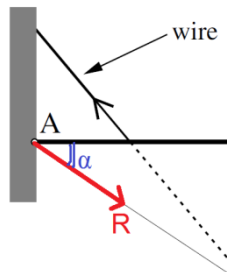
Resolve horizontally
$$X = T\cos 50 = 2088.65\cos 50$$
$$\therefore X = 1342.55...$$
$$\therefore X = 1343N$$
Resolve vertically
$$Y + 600 = T\sin 50 = 1600$$
$$\therefore Y = 1000N$$
Combine to find R
$$R = \sqrt{X^2 + Y^2} = \sqrt{1342.55^2 + 1000^2}$$
$$\therefore R = 1674.056...$$
$$\therefore R = 1674N$$
To find angle with the horizontal
$$\tan \alpha = \frac{Y}{X} = \frac{1600}{1342.55...}$$
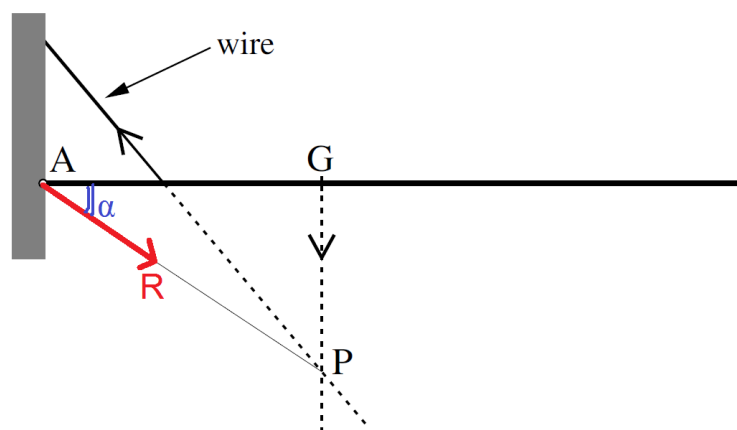$$\because \alpha = 36.68°$$

iv. Angle GAP

AP represents the line of action of the resultant which is at 36.68° to the horizontal
GP represents the line of action of the weight of beam
The wire through P represents the tension T

$$G\hat{A}P = \alpha = 36.68°$$
Since the beam is in equilibrium, all the forces T, W and R must pass through one concurrent point i.e. P as they are the only forces acting on the beam.

3i.	• With laminae it is a good idea to sketch each region or name by vertices
	• Remember when hanging by one point, the centre of mass is always vertically beneath that point
	• When a question asks to show, ensure you have sufficient method to ensure you give a convincing understanding
	• Centre of mass must be written as coordinates for last mark, do not leave as vectors

To show centre of mass (c.m.) is (0.8,2.5)

The mass of each strip is proportional to the area since the lamina is of uniform thickness
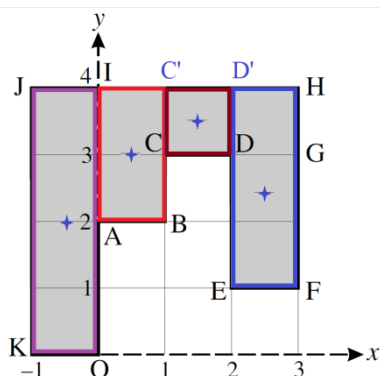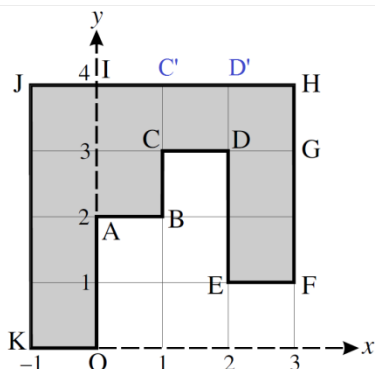
Taking vertical strips

	mass	Centre x	Centre y
OIJK	4x1=4	-0.5	2
ABC'I	2x1=2	0.5	3
CDD'C'	1x1=1	1.5	3.5
FHD'E	3x1=3	2.5	2.5
	Total=10		

To find centre of mass for whole lamina,

$$M\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\sum m\begin{pmatrix}x\\y\end{pmatrix}$$

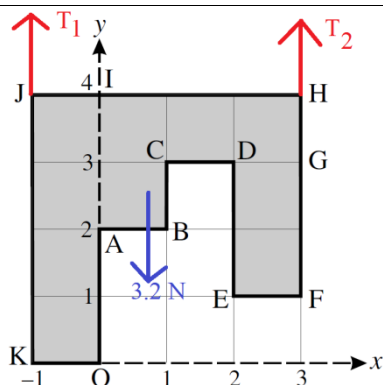$$\therefore 10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=4\begin{pmatrix}-0.5\\2\end{pmatrix}+2\begin{pmatrix}0.5\\3\end{pmatrix}+1\begin{pmatrix}1.5\\3.5\end{pmatrix}+3\begin{pmatrix}2.5\\2.5\end{pmatrix}$$

$$\therefore 10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}8\\25\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}0.8\\2.5\end{pmatrix}$$

The centre of mass is (0.8, 2.5)

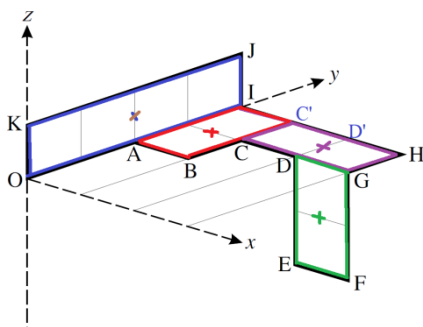| ii.		Tensions in wires at J and H are T_1 and T_2

Taking Moments about J

$$W\times 1.8 = T_2\times 4$$

$$\therefore T_2 = \frac{3.2\times 1.8}{4}=1.44N$$

Resolving vertically

$$T_1 + T_2 = 3.2$$

$$\therefore T_1 = 1.76N$$

iii.	To find centre of mass of lamina with folds	Taking vertical strips as shown

iii. To find centre of mass of lamina with folds

Taking vertical strips as shown

|	mass	Centre x	Centre y	Centre z
OIJK	4x1=4	0	2	0.5
ABC'I	2x1=2	0.5	3	0
CGHC'	1x2=2	2	3.5	0
FGDE	2x1=2	3.5	3	-1
	Total=10			

To find centre of mass for whole lamina,

$$M \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \sum m \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\therefore 10 \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = 4 \begin{pmatrix} 0 \\ 2 \\ 0.5 \end{pmatrix} + 2 \begin{pmatrix} 0.5 \\ 3 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 3.5 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 3.5 \\ 3 \\ -1 \end{pmatrix}$$

$$\therefore 10 \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{pmatrix} 10 \\ 27 \\ 0 \end{pmatrix} \quad \text{and so} \quad \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{pmatrix} 1 \\ 2.7 \\ 0 \end{pmatrix}$$

The centre of mass is (1, 2.7, 0)

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iv. Lamina hanging beneath O
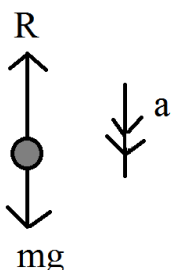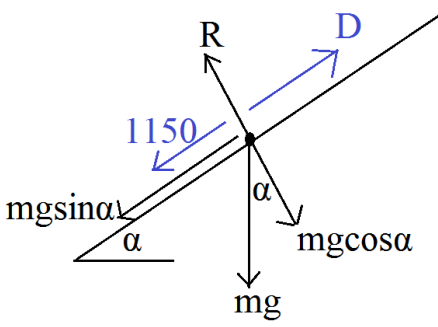Want angle OI makes with the vertical
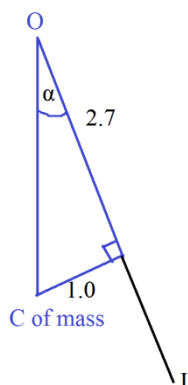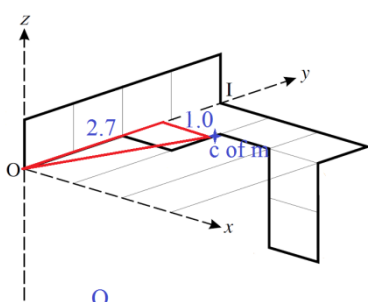
When the lamina is hanging from O the centre of mass will now be directly beneath O.

The centre of mass triangle lies on the xy plane to begin, so when hanging, this triangle will lie in the xz plane.

Let α be the angle between OI and the vertical then

$$\tan \alpha = \frac{1}{2.7}$$
$$\therefore \alpha = 20.3°$$

4(a)	Using energy methods and work done
	- Gravitational potential energy (gpe) = mgh Joules - Change in kinetic energy = $\dfrac{1}{2}m(v^2 - u^2)$ Joules - Impulse $I = Ft = m(v-u)$ Newton-seconds - Work done = Force x distance (in direction of force only) = Fs Joules - Power = Force x velocity Watts

	$m = 80kg$ $s = 1600$ $u = V$ $v = 6$ R ↑ ↓ a ● ↓ mg mg − R = ma So you get the same equation	Work done against resistances $=1.3 \times 10^6$ J Particle moving downwards, so loses potential energy, gains kinetic energy $$mgh = 80 \times 9.8 \times 1600$$ $$change\ ke = \frac{1}{2} \times 80(6^2 - V^2) = 1440 - 40V^2$$ $$\therefore 80 \times 9.8 \times 1600 - 1300000 = 1440 - 40V^2$$ $$\therefore V^2 = 1176$$ $$\therefore V = 34.3$$ The analogy with NLM2 would be using F=ma $$Rs = 1.3 \times 10^6 \Rightarrow R = \frac{1300000}{1600}$$ $$v^2 = u^2 + 2as \Rightarrow a = \frac{6^2 - V^2}{2 \times 1600}$$ $$mg - R = ma \Rightarrow 80 \times 9.8 - \frac{1300000}{1600} = 80 \times \left(\frac{6^2 - V^2}{2 \times 1600} \right)$$ $$\therefore 80 \times 9.8 \times 1600 - 1300000 = \frac{1}{2} \times 80 \times \left(6^2 - V^2\right)$$

(b)i.	$m = 800$ $\sin \alpha = 0.1$ $u = 8$ $a = 0.25$ R D 1150 α mgsinα mgcosα α mg	Resolving parallel to the plane using F=ma $D - 1150 - 800g \sin \alpha = 800 \times 0.25$ $\therefore D = 1150 + 784 + 200$ $\therefore D = 2134N$ Power = Force x velocity $P = 2134 \times 8$ Watts $\therefore P = 17072$

ii. | $\sin \alpha = 0.1$
$a = 0.25$
$m = 300$
$D = 900$
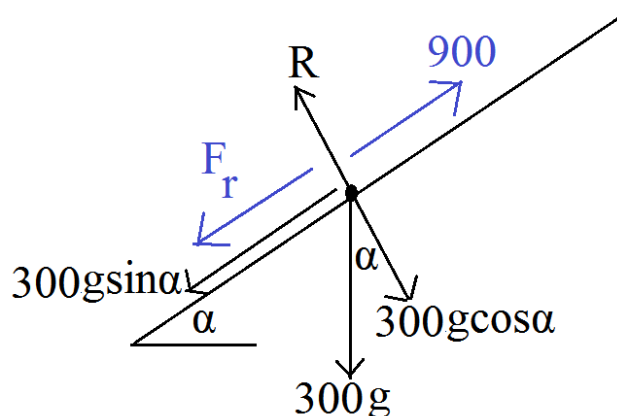
The sledge is connected to the vehicle, so it will still have the same acceleration as previously

Resolving perpendicular to the plane (a=0) using F=ma
$$R = 300g\cos\alpha = 300 \times 9.8 \times \cos(\sin^{-1}(0.1))$$
Resolving parallel to the plane (a=0.25) using F=ma
$$900 - F_r - 300g\sin\alpha = 300 \times 0.25$$
$$\therefore F_r = 900 - 294 - 75$$
$$\therefore F_r = 531$$
Using limiting friction
$$F_r \leq \mu R$$
$$\mu = \frac{F_r}{R} = \frac{531}{300g\cos\alpha} = 0.1815...$$
$$\therefore \mu = 0.182$$