MEI Mechanics 2 June 2009. Mrs. K. Wright.

1(a)

$u \rightarrow$ $\quad \leftarrow u$ before impact

(P) m $\quad$ (Q) km $\qquad \Rightarrow$ positive direction

(i)

after impact

$v \rightarrow$ $\quad \overrightarrow{\tfrac{1}{3}u}$

(ii) Conservation of linear momentum $\qquad$ Remember Q in opposite direction so negative initially

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\therefore \quad mu - kmu = mv + km\frac{u}{3}$$

$$\therefore \quad v = u - ku - \frac{ku}{3} = u - \frac{4}{3}ku$$

$$\therefore \quad v = \left(1 - \frac{4}{3}k\right)u.$$

(iii) If motion of P is reversed after the collision, v will be in opposite direction and therefore negative.

$$\therefore \quad 1 - \frac{4}{3}k < 0 \qquad \text{so} \qquad \frac{4}{3}k > 1 \qquad \text{and} \qquad k > \frac{3}{4}$$

(iv) If coefficient of restitution in collision is 0.5

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{v_2 - v_1}{u_1 - u_2} \qquad u_1 = u \quad v_1 = v \qquad u_2 = -u \quad v_2 = \frac{1}{3}u$$

$$\therefore \quad \frac{1}{3}u - v = e\left(u - (-u)\right)$$

$$\therefore \quad \frac{1}{3}u - \left(1 - \frac{4}{3}k\right)u = \frac{1}{2} \cdot 2u$$

$$\therefore \quad \frac{1}{3} - 1 + \frac{4}{3}k = 1$$

$$\therefore \quad k = \frac{5}{3} \times \frac{3}{4} = \frac{5}{4}$$

If $k = \frac{5}{4}$ then $v = \left(1 - \frac{4}{3} \times \frac{5}{4}\right)u = -\frac{2}{3}u.$

$$\therefore \quad v = -\frac{2}{3}u \quad \text{and} \quad k = \frac{5}{4}.$$

(b) $u = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ A, mass 5 $\qquad$ $u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ B, mass 3

(i) $F = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ for 9 seconds on B $\qquad$ using 'F = ma'

$\begin{pmatrix} 1 \\ -2 \end{pmatrix} = 3 \begin{pmatrix} a_x \\ a_y \end{pmatrix} \qquad \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} 1/3 \\ -2/3 \end{pmatrix}$

Using $v = u + at$ where $u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, a from above, $t = 9$

$\therefore V_B = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

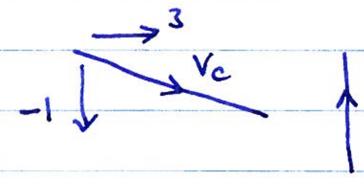
Using CLM $\qquad m_A u_A + m_B u_B = m_C u_C \qquad$ [before = after]

$\therefore 5 \begin{pmatrix} 3 \\ 2 \end{pmatrix} + 3 \begin{pmatrix} \; \\ \; \end{pmatrix} = 8 \begin{pmatrix} V_x \\ V_y \end{pmatrix}$

$\therefore 8 \begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} 24 \\ -8 \end{pmatrix}$

$\therefore V_c = \frac{1}{8} \begin{pmatrix} 24 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

(ii)

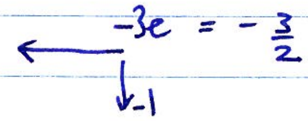


barrier is in direction $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ↑ so vertical

$e = \frac{1}{2}$

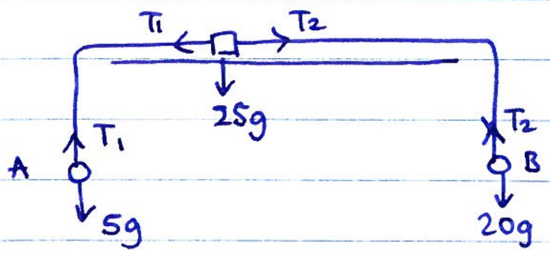
Only the perpendicular component is changed by the coefficient of restitution; the parallel component remains unaltered.

After impact



$\therefore V_{after} = \begin{pmatrix} -3/2 \\ -1 \end{pmatrix}$

2(a)



(i) Released from rest.

(A) Mechanical energy is preserved since table is smooth and no energy lost via friction so no work done; secondly gravity is not used on the block.

(B) No work is done by the block against reaction of table on it as table is smooth; motion is parallel to table surface.

(ii) $u=0$ $v=1.5$ ms^{-1}. at Point P.

Work done = force × distance (in direction of motion)

change in p.e. = mgh

B moves down as largest mass $\therefore$ mgh = 20gh − 5gh = 15gh

change in k.e. $= \frac{1}{2}m(v^2-u^2)$

Total mass in the system = 20 + 5 + 25 = 50

$\therefore$ k.e. $= \frac{1}{2} \times 50 \times (1.5^2 - 0^2)$

$\therefore$ $15gh = 25 \times 2.25$

$\therefore$ $h = 0.383$ metres.

so OP is 0.383 metres.

(iii) Beyond P, table has friction. PQ is 2 metres in which 25g block does 180 J work against resistance to motion. Speed at Q is v.

change in k.e. $= \frac{1}{2}m(v^2-u^2) = \frac{1}{2} \times 50 \times (v^2 - 1.5^2)$

Work done between P + Q = mgh for system − Friction work done

$\therefore$ change in p.e. $= 20g \times 2 - 5g \times 2 - 180$

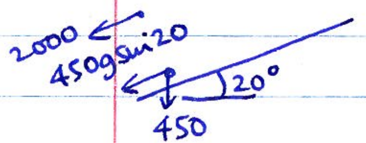
$\therefore$ $30g - 180 = 25(v^2 - 2.25)$

$\therefore$ $v^2 = 6.81$

$\therefore$ $v = 2.61$ ms^{-1}.

$\therefore$ speed at Q is 2.61 ms^{-1}.

(b) tree mass 450 kg slope 20° incline; steady speed = 2.5 ms^{-1}



$\therefore$ no acceleration. Frictional force = 2000 N

Power = Fv $= \frac{Fd}{t}$

Downward force = 450gsin20 + 2000

$\therefore$ $P = (450g\sin 20 + 2000) \times 2.5$

$\therefore$ $P = 8771$ Watts

$\therefore$ Power = 8770 Watts (3sf).

Q3.

(i)

Take moments about R_A.

CW 85×3

ACW $R_B \times 5$

$\therefore\ 5R_B = 3 \times 85$

$\therefore\ R_B = 51$ N.

Resolving vertically

$R_B + R_A = 85$

$\therefore\ R_A = 85 - 51$

$\therefore\ R_A = 34$ N.

or Taking moments about R_B

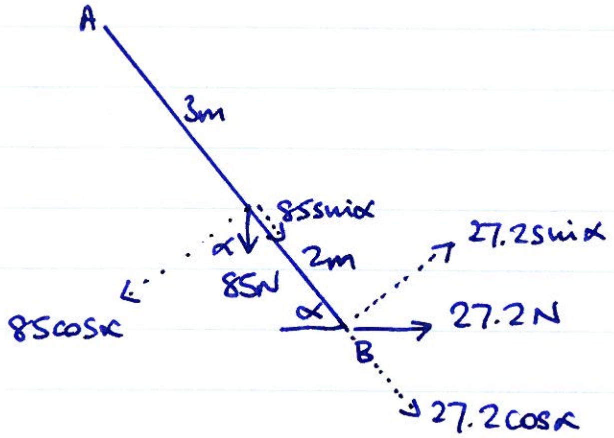
CW: $R_A \times 5$

ACW 85×2

$\therefore\ 5R_A = 2 \times 85$

$\therefore\ R_A = 34$ N.

(ii)



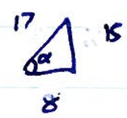
taking moments about A means we can ignore reaction force there. Resolving force at B and weight. Only forces perpendicular to beam produce moments about A.

Moments about A : ACW $27.2\sin\alpha \times 5$

CW $85\cos\alpha \times 3$

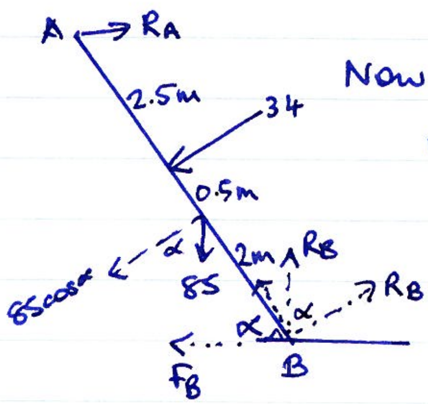
$\therefore\ 27.2\sin\alpha \times 5 = 85\cos\alpha \times 3$

$\therefore\ \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{85 \times 3}{27.2 \times 5}$



$\therefore\ \tan\alpha = \dfrac{15}{8}$ $\quad\therefore \cos\alpha = \dfrac{8}{17}$ and $\sin\alpha = \dfrac{15}{17}$

(iii)



Now resting on a rough floor and smooth wall – friction on the floor but not on the wall: on the point of slipping.

Could resolve as above

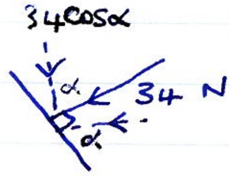
Better to take moments about A since we want to exclude those reaction forces

CW $\quad 34 \times 2.5 \quad + 85 \cos\alpha \times 3 \quad + F_B \sin\alpha \times 5$

ACW $\quad R_B \cos\alpha \times 5$

$\cos\alpha = \frac{8}{17}$

$\therefore \quad 205 + \frac{75}{17} F_B = \frac{40}{17} R_B$



$\sin\alpha = \frac{15}{17}$

Resolving vertically

$34\cos\alpha + 85 = R_B$

$\therefore R_B = 101$ N. $\qquad \therefore F_B = 7.4$ N

Resolving horizontally

$R_A = 34\sin\alpha + F_B$

$\therefore R_A = 37.4$ N

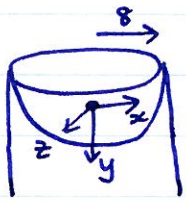
Given $F = \mu R$ for limiting friction

$$\mu = \frac{F}{R} = \frac{7.4}{101} = 0.073267\ldots$$

$\therefore \mu = 0.0733$ (3sf).

4.

(i)



part	mass (area)	x	y	z
bowl	$2\pi r^2 = 2\pi 64$	0	4	0
legs	$2\pi r h = \pi 16 k$	0	$k/2$	0
	$2\pi 64 + 16k\pi$			

$$M\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \sum m_i \begin{pmatrix}x_i\\ y_i\\ z_i\end{pmatrix}$$

$\therefore (2\pi 64 + 16k\pi)\,\bar{y} = 2\pi 64 \times 4 + 16k\pi \times \frac{k}{2}$

$(64 + 8k)\,\bar{y} = 256 + 4k^2 \qquad \div 2\pi$

$(16 + 2k)\,\bar{y} = 64 + k^2 \qquad \div 4$

$$\therefore \bar{y} = \frac{64 + k^2}{16 + 2k}$$

(ii) If $k = 12$, plus a circular end underneath the bowl

previously $\bar{y} = \dfrac{64 + k^2}{16 + 2k}$ so when $k = 12$, $\bar{y} = 5.2$

now

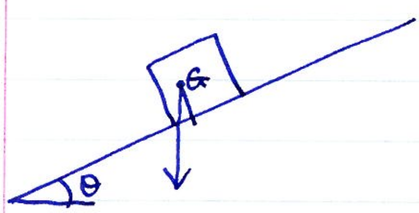
part	mass	y
previous	320π	5.2
end	64π	12

$\therefore \quad M\bar{y}_{new} = 320\pi \times 5.2 + 64\pi \times 12$

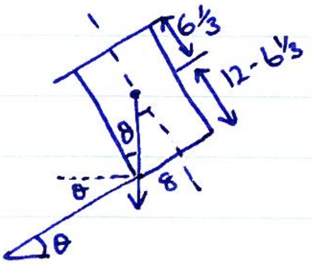
$\therefore \quad 384\pi \bar{y}_{new} = 1664\pi + 768\pi$

$\therefore \quad \bar{y}_{new} = \dfrac{1664 + 768}{384} = 6\frac{1}{3}$ down from O.

(iii)



Not sliding and on point of toppling. Centre of mass directly over edge of tipping

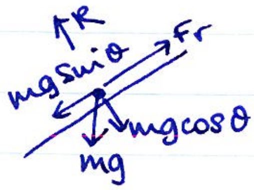


$\tan\theta = \dfrac{8}{(12 - 6\frac{1}{3})} = \dfrac{8}{(5\frac{2}{3})} = \dfrac{24}{17}$

$\therefore \quad \theta = \tan^{-1}\left(\dfrac{24}{17}\right) = 54.6887\ldots$

$\therefore \quad \theta = 54.7^\circ$ (3sf)

(iv) If $\mu = 1.5$ will bowl still slide?



$\mu = \dfrac{F}{R} \qquad F \leq \mu R$

$R = mg\cos\theta \qquad F = 1.5 \times mg\cos\theta$

$mg\sin\theta - Fr = mg\sin\theta - 1.5mg\cos\theta = -0.05100\ldots$

so $mg\sin\theta - Fr < 0$ implies will not slide.

or $\mu = \dfrac{F}{R} = \dfrac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \dfrac{24}{17} = 1.41176\ldots$

Since given $\mu = 1.5 > 1.411\ldots$ then will not slide.