M2 June 2008. Mrs K Wright

1(a)

$\rightarrow u_A$ $\qquad\qquad$ $\rightarrow u_B$

(A) $\qquad\qquad$ (B) $\qquad\qquad$ $\rightarrow i$

$m = 6$ $\qquad\qquad$ $m = 0.5$

$\rightarrow 3i$ $\qquad\qquad$ $\rightarrow 11i$

Impulse on A in collision $= -12i$ Ns

After collision, velocity of A $= 3i$

velocity of B $= 11i$

(i) using conservation of linear momentum

$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$

$$\therefore \quad 6u_a + 0.5u_b = 6(3i) + 0.5(11i)$$

since $-6u_a + 6(3i)$ is change in momentum $(mv - mu)$

$$18i - 6(u_a) = -12i$$

$$\therefore \quad 30i = 6u_a$$

$$\therefore u_a = 5i$$

$$\therefore \quad 6(5i) + 0.5(u_b) = 18i + 5.5i$$

$$\therefore 0.5(u_b) = -6.5i$$

$$\therefore u_b = -13i$$

(ii)
$$e = \frac{v_b - v_a}{u_a - u_b} = \frac{11i - 3i}{5i - (-13i)} = \frac{8i}{18i} = \frac{4}{9}$$

$\therefore \quad e = \frac{4}{9}$ this is the coefficient of restitution

(iii) A force of $-2i$ N acts on B for 7 seconds after the collision.

using $u = 11i$ after collision and $F = ma$

$$-2i = 0.5a$$

$$\therefore a = -4i$$

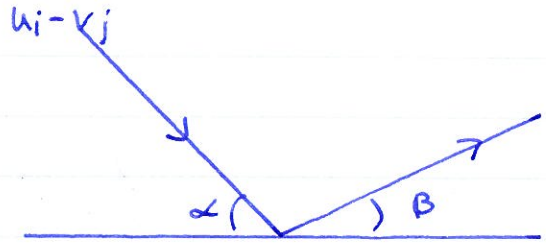
using $v = u + at$

$$v = 11i + (-4i) \times 7$$

$$\therefore v = 11i - 28i$$

$$\therefore v = -17i \quad \text{m/s.}$$

1 (b)



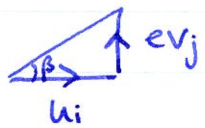
The coefficient e affects only the component which is perpendicular to the surface which the balls bounce off.

If velocity before collision $= -vj$

then velocity after collision $= evj$

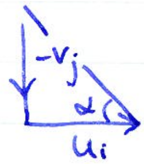
$\therefore$ velocity resultant after collision $= ui + evj$

After:



$$\tan\beta = \frac{evj}{ui}$$

since before

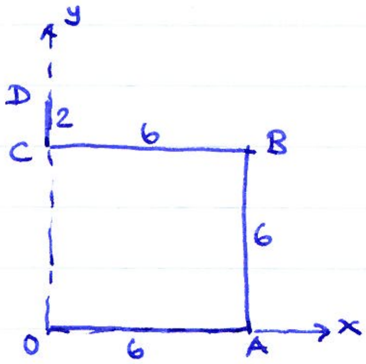


$$\tan\alpha = \frac{vj}{ui}$$

$\therefore \tan\beta = e\tan\alpha.$

2.

(i)



(i)

part	mass	x	y
OA	6	3	0
AB	6	6	3
BC	6	3	6
CD	2	0	7
	20		

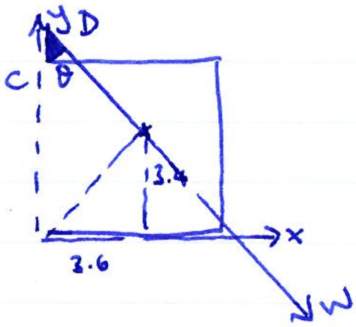
$M\bar{x} = \sum m_i x_i$

$$\therefore \quad 20\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 6\begin{pmatrix}3\\0\end{pmatrix} + 6\begin{pmatrix}6\\3\end{pmatrix} + 6\begin{pmatrix}3\\6\end{pmatrix} + 2\begin{pmatrix}0\\7\end{pmatrix} = \begin{pmatrix}72\\68\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}3.6\\3.4\end{pmatrix}$$

$\therefore$ centre of mass $(\bar{x}, \bar{y}) = (3.6, 3.4)$ cm from O

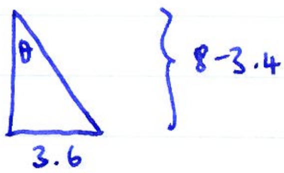
(ii)



Now hanging from D, and centre of mass lies directly beneath D

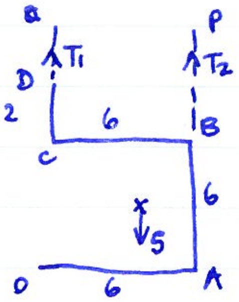
angle of CD to the vertical

(iii)



$\tan\theta = \dfrac{3.6}{4.6}$

$\therefore \theta = 38.0°$



$T_1 + T_2 = 5$ (Resolving vertically

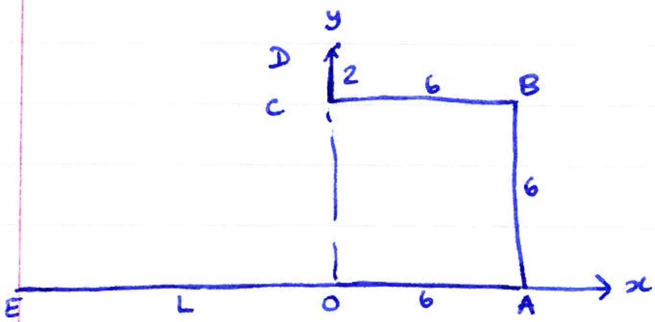
Taking moments about Q

$T_2 \times 6 = W \times 3.6$

Since $W = 5$ $T_2 = 3$ N

$\therefore T_1 = 2$ N

(iv)



centre of mass on the y axis

part	mass	x	y
original	20	3.6	3.4
new	L	$-\frac{L}{2}$	0
	20+L	0	$\bar{y}$

$$\therefore (20+L)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 20\begin{pmatrix}3.6\\ 3.4\end{pmatrix} + L\begin{pmatrix}-\frac{L}{2}\\ 0\end{pmatrix}$$

but $\bar{x} = 0$

$\therefore 72 - \dfrac{L^2}{2} = 0$

$\therefore L^2 = 144$

$\therefore L = 12$

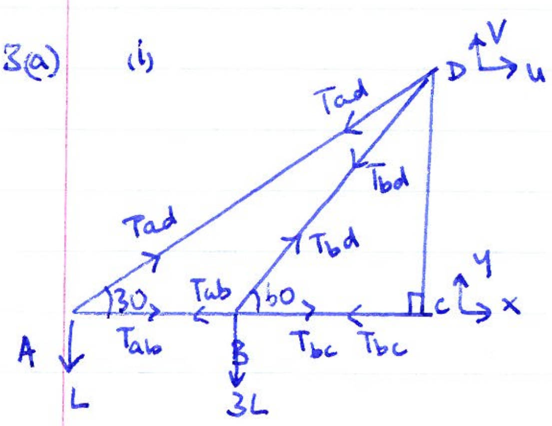
For $\bar{y}$

$(20+12)\bar{y} = 68$

$\therefore \bar{y} = \dfrac{68}{32} = 2.125$.

$\therefore$ length OE must be 12 cm.

3(a) (i)



(ii) At A : vertically $T_{ad}\sin 30 = L$

$\therefore T_{ad} = 2L$ (tension)

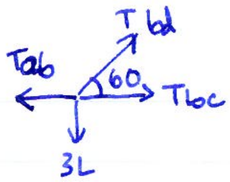
horizontally

$T_{ad}\cos 30 + T_{ab} = 0$

$\therefore 2L \times \dfrac{\sqrt{3}}{2} + T_{ab} = 0$

$\therefore T_{ab} = -\sqrt{3}L$ (compression)

3(ii)



At B: Vertically $T_{bd} \sin 60 = 3L$

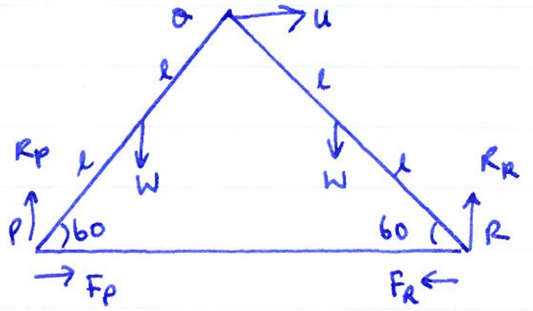
$$\therefore T_{bd} = \frac{6L}{\sqrt{3}} = 2\sqrt{3}\,L \quad \text{(tension)}$$

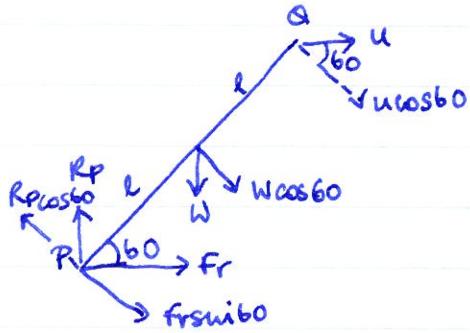
Horizontally $T_{ab} = T_{bc} + T_{bd}\cos 60$

$$\therefore -\sqrt{3}\,L = T_{bc} + 2\sqrt{3}\,L \times \tfrac{1}{2}$$

$$\therefore T_{bc} = -2\sqrt{3}\,L \quad \text{(compression)}$$

3(b)





Since in equilibrium, moments about Q

$$R_p\cos 60 \times 2l = F_r \sin 60 \times 2l + W\cos 60 \times l$$

$$R_p = F_r\sqrt{3} + \frac{W}{2}$$

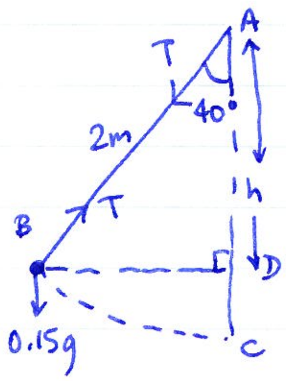
Resolving vertically $R_p = W$

$$\therefore W = F_r\sqrt{3} + \frac{W}{2}$$

$$\therefore \frac{W}{2} = F_r\sqrt{3}$$

$$\therefore F_r = \frac{W}{2\sqrt{3}} = \frac{W\sqrt{3}}{6}$$

Q4(a)



(i) no work is done by T, the tension in the line because it is not acting in the direction of motion

(ii) $\cos 40 = \frac{h}{2}$ $\therefore h = 2\cos 40$

AC = 2m (radius)

$\therefore$ drop in height DC is $2 - 2\cos 40 = 0.46791\ldots$

$= 0.4679$ m (4dp)

loss in gravitational energy $= mgh$

$= 0.15 \times 9.8 \times 0.4679\ldots$

$= 0.688$ Joules (3sf)

4 (iii) If there is no air resistance, speed at start $u=0$, and at $C = v$

$\therefore$ work done = change in kinetic energy

$\therefore \quad mgh = \frac{1}{2}m(v^2-u^2)$

$\therefore \quad v^2 = 9.17105\ldots$

$\therefore v = 3.03$ m/s

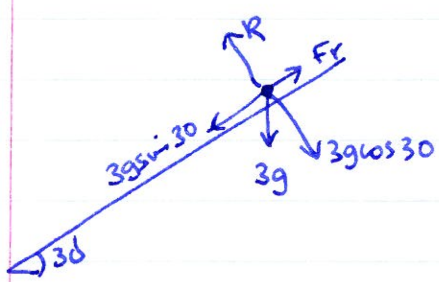
(iv) If loss of 0.6 J for every metre travelled $\left[= \frac{40}{360} \times 2\pi \times 2\right] = \frac{4}{9}\pi \times 0.6$ J

sphere has initial speed of 2.5 m/s

$\therefore \quad mgh - \frac{4}{15}\pi = \frac{1}{2}m(v^2 - 2.5^2)$

$\therefore v^2 = 4.25095\ldots$

$\therefore v = 2.06$ m/s

(b) 

$a = \frac{1}{8}g \qquad R = 3g\cos 30$ (Resolving ⊥ to plane)

"F=ma" down plane

$3g\sin 30 - Fr = 3 \times \frac{1}{8}g$

$\therefore Fr = \frac{3}{2}g - \frac{3}{8}g = 11.025 = \frac{9g}{8}$

For limiting friction $F \leq \mu R$ so $\mu \geq \frac{F}{R}$

$\therefore \mu \geq \frac{11.025}{3g\cos 30} = \frac{\frac{9g}{8}}{3g\frac{\sqrt{3}}{2}} = \frac{3 \times 2}{8\sqrt{3}} = \frac{\sqrt{3}}{4}$

$\therefore \mu = \frac{\sqrt{3}}{4}$