M2 June 2007 Mrs K Wright.

1. (a) (i)

$u=0$ $\rightarrow F=9$ $t=2$

○

$m=6$

Impulse $= mv - mu = Ft$

$\therefore I = 9 \times 2 = 18$

$\therefore 6v - 0 = 18$ ~~$v - 0 = 3$~~

$\therefore v = 3$ m/s

(ii)

$u=3 \rightarrow$ (A) $m=6$

$\leftarrow u=-1$ (B) $m=2$

coalesce after impact

Using conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\therefore 6(3) + 2(-1) = (6+2)v$$

$$\therefore v = 2 \text{ m/s}$$

So velocity after collision is 2m/s in direction of initial motion of particle A.

(iii) Impulse acting on B $= mv - mu$

$= 2 \times 2 - 2(-1)$

$= 6$ Ns.

(iv) (A)

$2 \rightarrow$ (AB) $M=8$ $\rightarrow v_1$

$\rightarrow 1.8$ (C) $M=10$ $\rightarrow v_2 = 1.9$

(B)

$$8(2) + 10(1.8) = 8v_1 + 10 \times 1.9$$

$$\therefore v_1 = \frac{15}{8} = 1.875 \text{ m/s}$$

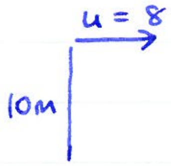
$$\therefore v_1 = 1.88 \text{ m/s} \quad (3sf)$$

(c) Coefficient of restitution $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$$\therefore e = \frac{1.9 - 1.875}{2 - 1.8} = \frac{0.025}{0.2}$$

$$\therefore e = 0.125$$

1 (b)



$e = \frac{4}{7}$

horizontal	vertical
$u = 8$	$u = 0$
v	$v =$
$a = 0$	$a = 9.8$
s	$s = 10$
t	t

vertical component before hitting the ground

using $v^2 = u^2 + 2as$

$v^2 = 0 + 19.6 \times 10$

$\therefore v^2 = 196$

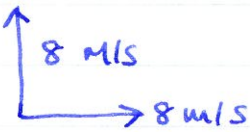
$\therefore v = 14$ m/s

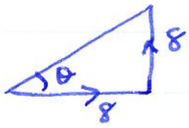
After bounce, using $e = \frac{4}{7}$ new velocity $= 14 \times \frac{4}{7} = 8$ m/s up.

($\therefore V_a = -8$ m/s based on +ve being down)

Since coefficient of restitution affects only the perpendicular component of velocity, the parallel component remains the same

so after bounce



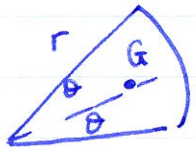


$\tan\theta = \frac{8}{8} = 1$

$\therefore \theta = 45°$

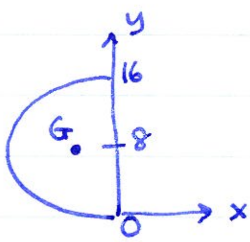
So bounces at 45° to the plane.

Q2 (i)



$CG = \frac{r\sin\theta}{\theta}$ where θ in radians

If wire looks like



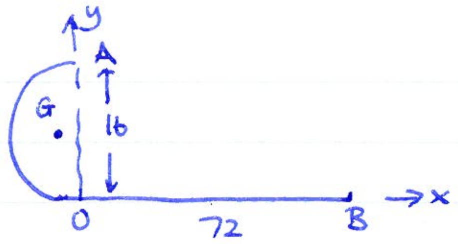
using $\theta = \frac{\pi}{2}$ and $r = 8$

$\frac{r\sin\theta}{\theta} = \frac{8}{\pi/2} = \frac{16}{\pi}$

So centre of mass G is $\frac{16}{\pi}$ in x direction and 8 in y

$\therefore$ c of m $\left(-\frac{16}{\pi}, 8\right)$

(ii)



part	mass	x	y
curve	$\pi r = \pi 8$	$-\frac{16}{\pi}$	8
length	72	36	0
	$8\pi + 72$		

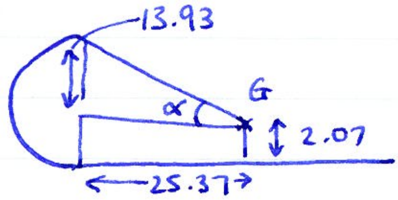
$\therefore \; M\bar{x} = \sum m_i x_i$

$\therefore \; (8\pi + 72)\begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix} = 8\pi\begin{pmatrix}-16/\pi \\ 8\end{pmatrix} + 72\begin{pmatrix}36 \\ 0\end{pmatrix} = \begin{pmatrix}2464 \\ 64\pi\end{pmatrix}$

$\therefore \; \begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix} = \begin{pmatrix}25.37 \\ 2.07\end{pmatrix}$ (2dp)

$\therefore$ c of m $(\bar{x}, \bar{y})$ is $(25.37, 2.07)$ to 2dp.

(iii) When hung from a shelf from point A, the centre of mass will lie directly beneath A.



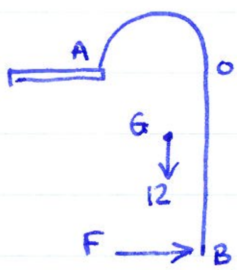
$16 - 2.07 \sim 13.93$

$\tan\alpha = \dfrac{13.93}{25.37}$

$\therefore \; \alpha = 28.77°$

$\therefore \; \alpha = 28.8°$ (3sf)

(iv)

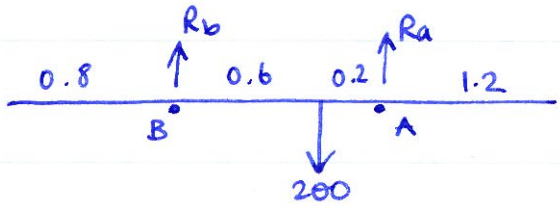


Taking moments about A

$12 \times 13.93 = F \times 16$

$\therefore \; F = 10.45$ N

3. (i)



Resolving vertically

$R_a + R_b = 200$

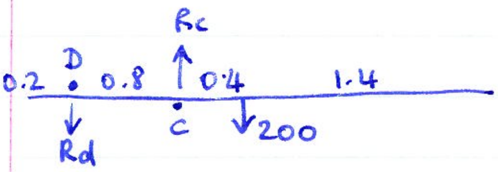
Moments about B

$R_a \times 0.8 = 200 \times 0.2$

$\therefore \; R_a = 50$ N

$R_b = 150$ N

(ii)



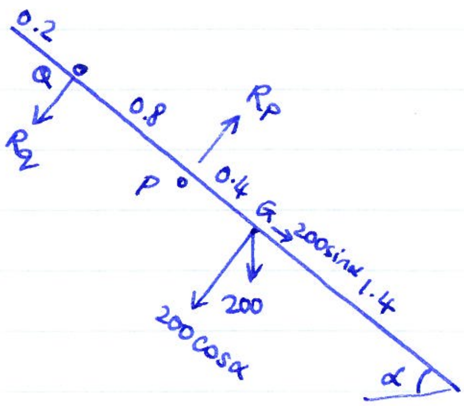
$R_c = R_d + 200$ (vertical equilibrium)

$200 \times 0.4 = R_d \times 0.8$ (moments about C)

$R_d = 100$ N

$R_c = 300$ N.

(iii)



$\sin\alpha = 0.28$

$\cos\alpha = 0.96$

Equilibrium ; resolve perpendicular to the plank

$R_q + 200\cos\alpha = R_P$

Moments about P

$R_q \times 0.8 = 200\cos\alpha \times 0.4$

$\therefore R_q = 100 \times 0.96 = 96$ N

$\therefore R_q \rightarrow 96 + 200 \times 0.96 = R_P$

$\therefore R_P = 288$ N

(iv) If only one peg is rough ?
Want smallest coefficient of friction possible.

For limiting friction $Fr \leq \mu R$

$\mu \geq \frac{Fr}{R}$ where $Fr = 200\sin\alpha = 200 \times 0.28 = 56$ N

For least μ, need to have greatest denominator.

$R_P > R_q$

$\therefore \mu = \frac{56}{288} = 0.194$ (3sf)

4. (i)

work done to raise mass from $u=0$ to $v=0.5$ in distance $= 4$m

Work done $= mgh = 20g \times 4 = 80g$ (gpe)

kinetic energy $= \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 20 \times 0.25 = 2.5$ J

$\therefore$ work done $= 80g + 2.5$

$= 786.5$ Joules

check with SUVAT $v^2 = u^2 + 2as$ $\therefore a = \frac{0.25}{8} = \frac{1}{32}$ m/s²

$\therefore T - 20g = 20 \times \frac{1}{32}$ using F=ma upwards

$\therefore T = 196.625$

Work done = force × distance $= 196.625 \times 4$

$= 786.5$ Joules.

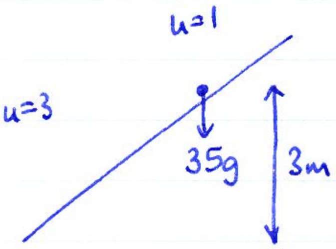
4 (ii) Power $= \dfrac{\text{work done}}{\text{time}} = \dfrac{Fd}{t} = Fv$ when at a steady speed, no acceleration

Upwards force $= T = mg$, $v = 0.5$

$\therefore P = 20g \times 0.5$

$\therefore P = 98$ Watt.

(iii)



work done due to gravitational potential energy $= mgh$

$= 35g \times 3$

$= 105g$ Joules

change in kinetic energy

$= \frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2} \times 35\ (9-1)$

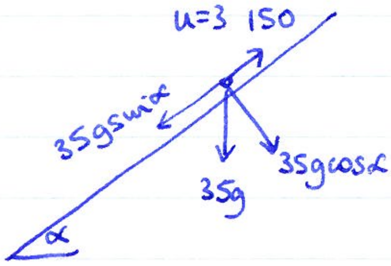
$= 140$ Joules

$\therefore$ $105g$ − work done by friction $= 140$

$\therefore$ work done by friction $= 1029 - 140$

$= 889$ Joules.

(iv)



$\sin\alpha = 0.1$

using "F=ma" down the slope

$35g\sin\alpha - 150 = 35a$

$\therefore a = -3.3057....$ m/s²

using $v^2 = u^2 + 2as$

$\therefore 0 = 9 + 2(-3.3057...)s$

$\therefore s = 1.3613$ metres.