M2 June 2006 Mrs K Wright.

1.

(i) Spheres coalesce after collision $4 \rightarrow \quad \leftarrow 2$

(A) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $\qquad v \rightarrow$

$\therefore \; 2(4) + 6(-2) = 8v$

$\therefore \quad v = \frac{-4}{8} = \frac{-1}{2}$

so motion after collision speed 0.5 m/s in direction of Q initial.

(B) Energy lost in the collision $= \frac{1}{2}m(v^2 - u^2)$

ke before $= \frac{1}{2} \times 2 \times 4^2 + \frac{1}{2} \times 6 \times (-2)^2 = 28$ J.

ke after $= \frac{1}{2} \times 8 \times \left(\frac{-1}{2}\right)^2 = 1$

$\therefore$ energy lost $= 27$ Joules.

(ii) Rebound with coefficient of restitution $e = \frac{2}{3}$ $\qquad v_1 \rightarrow \quad v_2 \rightarrow$ after

(A) CLM $\quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$2(4) + 6(-2) = 2(v_1) + 6(v_2)$

Restitution $\quad e = \frac{v_2 - v_1}{u_1 - u_2} \quad \therefore \; \frac{2}{3}(u_1 - u_2) = (v_2 - v_1)$

$\therefore \; 2(4) - 2(-2) = 3v_2 - 3v_1$

① $\quad 2v_1 + 6v_2 = -4$

② $\quad 3v_2 - 3v_1 = 12$

$\quad 6v_2 - 6v_1 = 24$

$\therefore 8v_1 = -28$

$\therefore v_1 = -3.5$ m/s $\quad$ velocity of P after collision

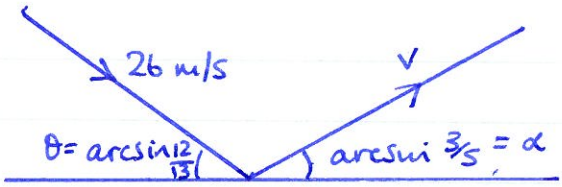
$\therefore v_2 = 0.5$ m/s $\quad$ velocity of Q after collision

(B) Impulse $= Ft = mv - mu$

For P : $\quad I = 2(-3.5 - 4) = -15$ Ns

This is an impulse of 15 Ns in direction of Q initial velocity.

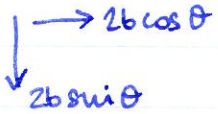
1(b)

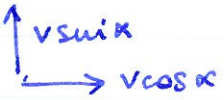



$\therefore \sin\theta = \frac{12}{13}$ $\quad\quad$ $\therefore \sin\alpha = \frac{3}{5}$

Right angle triangle 5, 12, 13 $\quad\quad$ Right angle triangle 3, 4, 5

$\therefore \cos\theta = \frac{5}{13}$ $\quad\quad$ $\therefore \cos\alpha = \frac{4}{5}$

before 


after 


Only the component perpendicular to the surface is changed by the coefficient of restitution.

The parallel component remains unchanged.

$e = \frac{v_2 - v_1}{u_1 - u_2}$ but $u_2, v_2 = 0$ for the surface so

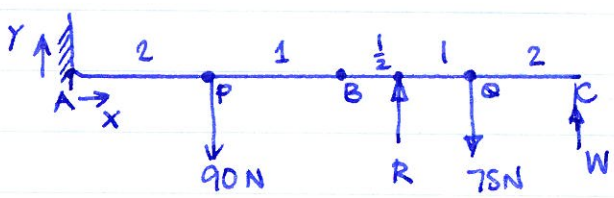
$e = -\frac{v_1}{u_1} = \frac{v\sin\alpha}{26\sin\theta}$

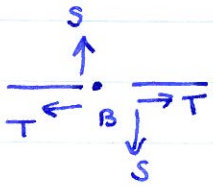
and $26\cos\theta = v\cos\alpha$

$\therefore 26 \times \frac{5}{13} = v \times \frac{4}{5} \Rightarrow \therefore v = \frac{50}{4} = 12.5$ m/s

and $e = \frac{\frac{50}{4} \times \frac{3}{5}}{26 \times \frac{12}{13}} = \frac{30}{4 \times 24} = \frac{5}{16} \Rightarrow \therefore e = \frac{5}{16}$.

Q2.




at B 


(i) Taking moments about A for rod AB and About R for BC

$2 \times 90 = 3 \times S$

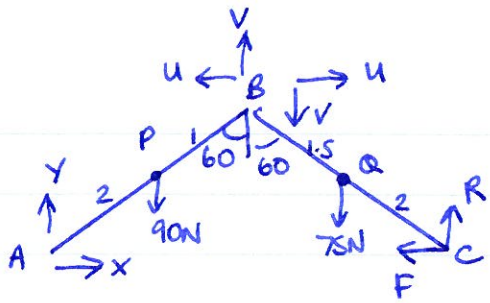
$\therefore S = 60$ N

$W \times 3 + S \times \frac{1}{2} = 75 \times 1$

$\therefore 3W = 75 - 30 = 45$

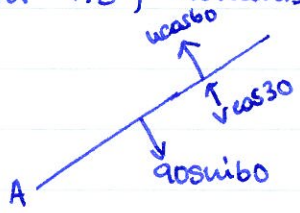
$\therefore W = 15$ N.

(ii)




The rods are in equilibrium and C on rough table.

for AB, moments about A

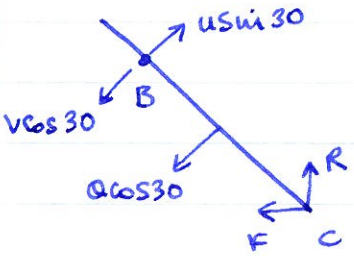



$90\sin 60 \times 2 = u\cos 60 \times 3 + v\cos 30 \times 3$

$\therefore 90 \times \frac{\sqrt{3}}{2} \times 2 = u \times \frac{1}{2} \times 3 + v \times \frac{\sqrt{3}}{2} \times 3$

$\therefore 60\sqrt{3} = u + v\sqrt{3}$ .

(iii)




(iv) Taking moments about C

$75\cos 30 \times 2 + v\cos 30 \times 3.5 = u\sin 30 \times 3.5$

$\therefore 75 \times \frac{\sqrt{3}}{2} \times 2 + v \times \frac{\sqrt{3}}{2} \times 3.5 = u \times \frac{1}{2} \times 3.5$

$\therefore 300\sqrt{3} + 7v\sqrt{3} = 7u$

from equations ① and ② $300\sqrt{3} = 7u - 7v\sqrt{3}$

② +7①

$420\sqrt{3} = 7u + 7v\sqrt{3}$

$300\sqrt{3} = 7u - 7v\sqrt{3}$

$720\sqrt{3} = 14u$

$\therefore u = \frac{360\sqrt{3}}{7} = 89.1$ N (3sf)

and $v\sqrt{3} = 60\sqrt{3} - \frac{360\sqrt{3}}{7} = (420 - 360)\frac{\sqrt{3}}{7}$

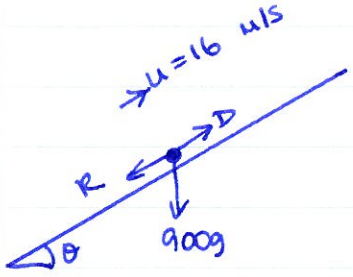
$\therefore v = \frac{60}{7} = 8.57$ N (3sf)

Resolve horizontally for friction F

$\therefore F = u$

$\therefore F = 89.1$ N (3sf)

3(a)




Power = fv  $v = 16$ m/s

$P = 20$ KW $= 20000$ Watts

$20000 = F \times 16$

$\therefore F = 1250$ N

So total force overcoming resistance and weight

$\therefore$ Weight produces downwards component parallel to slope

$= 900g \sin\theta$

$= 900g \times 0.1$

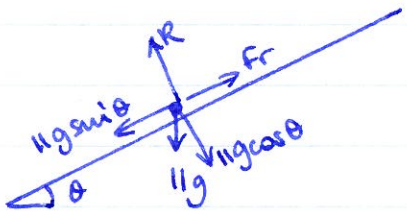
$= 90g$

$= 882$ N.

$\therefore$ Resistive forces total $= 1250 - 882$

$= 368$ N.

(b)




$\theta = \arccos \frac{12}{13}$  $\therefore \cos\theta = \frac{12}{13}$

(5, 12, 13)  $\therefore \sin\theta = \frac{5}{13}$

$\mu$ between slope & box

(i) If box stays at rest then

$R = 11g\cos\theta = 11g \times \frac{12}{13} = \frac{132g}{13}$

$F_r = 11g\sin\theta = 11g \times \frac{5}{13} = \frac{55g}{13}$

Limiting Equilibrium  $F \leq \mu R$

$\therefore \mu \geq \frac{F}{R}$ , $\mu \geq \frac{55g}{132g}$ , $\mu \geq \frac{5}{12}$

$\therefore \mu \geq \frac{5}{12}$

(ii) $\mu = 0.2$  Work done against air resistance = 6 joules per metre.

$x = 1.5$ metres up slope,  w.d. against AR $= 6 \times 1.5 = 9$ J.

$R = 11g\cos\theta$

$F = \mu R = 0.2 \times 11g \times \frac{12}{13}$

$\therefore F = \frac{132g}{65}$

using work done up the slope = change in kinetic energy

wd against AR + wd against friction + wd against weight = $\frac{1}{2}m(v^2-u^2)$

$$9 + \frac{132g}{65} \times 1.5 + 11g\sin\theta \times 1.5 = \frac{1}{2} \times 11 \times v^2$$

$\therefore \quad v^2 = 18.37\ldots$

$\therefore \quad v = 4.29$ m/s up the slope

Could have used $F=ma$

$$-6 - \frac{132g}{65} - 11g\sin\theta = 11a$$

$\therefore a = -6.12$ ms$^{-2}$ (negative as slows down)

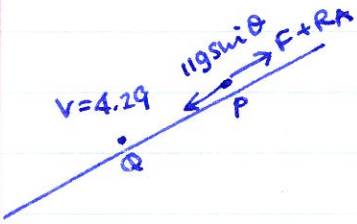
$\therefore$ "$v^2 = u^2 + 2as$" $u = V_1$, $s = 1.5$, $v = 0$

$\therefore v^2 + 3 \times (-6.12) = 0$

$\therefore v = 4.29$ m/s.

* watch strange use of variables.

(iii) box slides back down, speed of original projection now at Q




Downwards

$11g\sin\theta - F - R_A = 11a$ (NLM2)

or Energy method let distance from rest to Q be $x$ metres (rest to P is 1.5 metres as before)

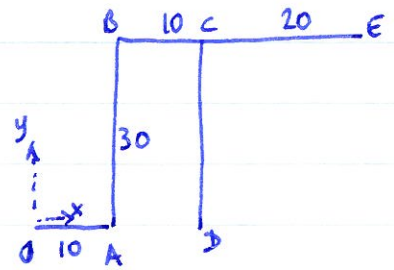
$\therefore 11g\sin\theta \, x - Fx - R_A x = \frac{1}{2}m(v^2-u^2)$

$$x\left(\frac{55g}{13} - \frac{132g}{65} - 6\right) = \frac{1}{2} \times 11 \times 4.286^2$$

$\therefore x = 6.49$ metres.

However, from P to Q is $6.49 - 1.5 = 4.99$ metres.

4. (i)




| | | Centre of mass |
|---|---|---|
| OA | 10 | 5, 0 |
| AB | 30 | 10, 15 |
| BC | 10 | 15, 30 |
| CD | 30 | 20, 15 |
| CE | 20 | 30, 30 |
| | 100 | $\bar{x}$ $\bar{y}$ |

Since $M\bar{x} = \Sigma m_i x_i$

$$100\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 10\begin{pmatrix}5\\ 0\end{pmatrix} + 30\begin{pmatrix}10\\ 15\end{pmatrix} + 10\begin{pmatrix}15\\ 30\end{pmatrix} + 30\begin{pmatrix}20\\ 15\end{pmatrix} + 20\begin{pmatrix}30\\ 30\end{pmatrix} = \begin{pmatrix}1700\\ 1800\end{pmatrix}$$

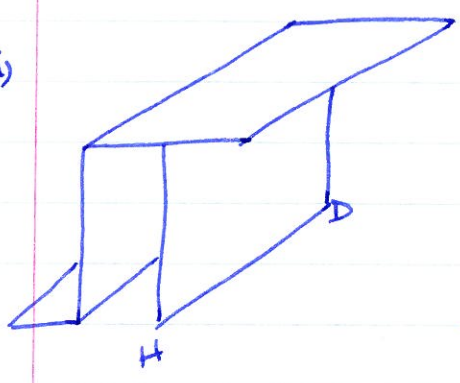
$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}17\\ 18\end{pmatrix}$$

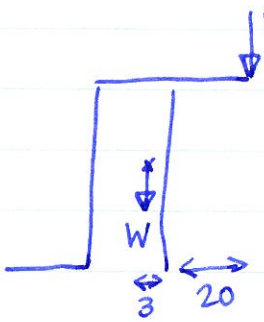
The centre of mass is (17, 18)

(ii) The 3D lamina model has similar centre of mass, but also a z coordinate of half its length

com = (17, 18, 20)

(iii)



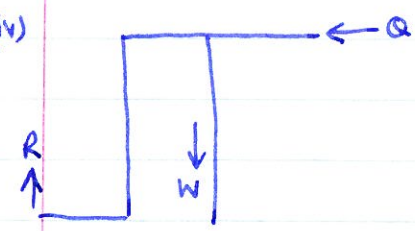

When bracket is on the point of tipping about edge HD, moments are in equilibrium

$\therefore W \times 3 = P \times 20$

$W = 60\ N$

$\therefore P = 9\ N.$

(iv)




(v) Moments about $O_z$

$Q \times 30 = W \times 17$

$\therefore Q = \frac{60 \times 17}{30}$

$\therefore Q = 34\ N$

So will tip over when Q reaches 34 N

If the frictional force is 30N < 34 N the bracket will slide before tipping over.