M2. January 2009

1. particle mass m kg from rest to $2u$ ms^{-1} in 5 seconds?

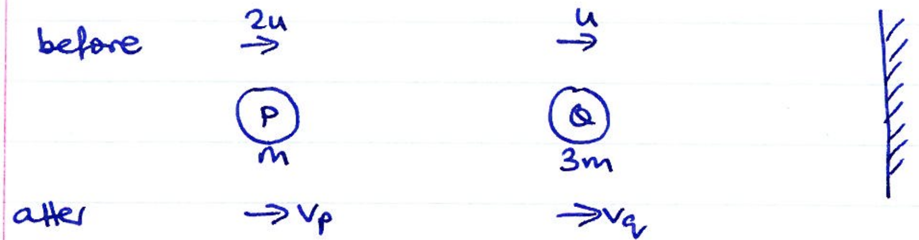
(i) using "$F = ma$" and "$v = u + at$"

$$2u = 0 + 5a$$

$$\therefore a = \frac{2u}{5} \quad (ms^{-2})$$

$$\therefore F = m\frac{2}{5}u = \frac{2}{5}mu. \quad (N)$$

(ii)



For perfectly elastic collision ($e=1$) using conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\therefore m(2u) + 3m(u) = mV_p + 3mV_q$$

$$\therefore V_p + 3V_q = 5u \quad (1)$$

for $e = \frac{V_2 - V_1}{u_1 - u_2}$ $\quad \therefore 1 = \frac{V_q - V_p}{2u - u}$

$$\therefore V_q - V_p = u \quad (2)$$

using (1) and (2) $\quad 4V_q = 6u$

$$\therefore V_q = \frac{3}{2}u.$$

$$\therefore V_p = V_q - u$$

$$\therefore V_p = \frac{1}{2}u$$

(iii) Q hits barrier and returns with a speed $= -eV_q = -\frac{3}{2}eu$

$$\therefore m\left(\tfrac{1}{2}u\right) + 3m\left(-\tfrac{3}{2}eu\right) = 4m\left(-\tfrac{1}{4}u\right)$$

$\therefore \frac{1}{2}u - \frac{9}{2}eu = -u$

$\therefore \frac{9}{2}eu = \frac{3}{2}u$

$\therefore e = \frac{1}{3}.$

Impulse = change in momentum

$I = mv - mu$

$\rightarrow \frac{3u}{2}$

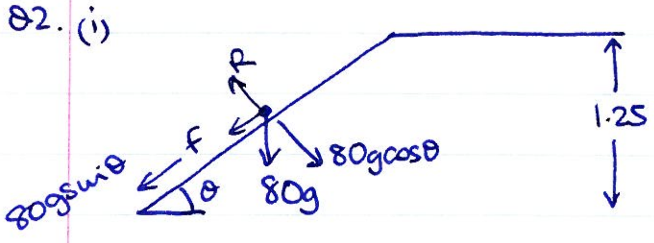
$\leftarrow -\frac{3}{2}eu = -\frac{3}{2} \times \frac{1}{3}u = -\frac{1}{2}u$

For Q $\quad I = 3m\left(\frac{1}{2}u - -\frac{3u}{2}\right)$

$\therefore I = 3m\left(\frac{1}{2}u + \frac{3u}{2}\right) = 6mu$

For barrier impulse = 6mu.

Q2. (i)



$\mu = 0.4$

$F \leq \mu R$

$R = 80g\cos\theta$

$\therefore F = 0.4 \times 80g\cos\theta$

$\therefore F = 32g\cos\theta$

(ii) distance up the ramp $\quad \sin\theta = \frac{1.25}{d} \quad \therefore d = \frac{1.25}{\sin\theta}$

work done against friction $= Fd$

$= 32g\cos\theta \times \frac{1.25}{\sin\theta}$

$= \frac{392}{\tan\theta}$

(iii) gain in gravitational potential energy = mgh

$= 80g \times 1.25$

= 980 Joules.

(iv) when $\theta = 35^\circ$

Power required to slide the box up the ramp at steady speed of $1.5\ ms^{-1}$.

$P = Fv = F\frac{d}{t} = 32g\cos 35 \times 1.5 = 385.33$ Watts

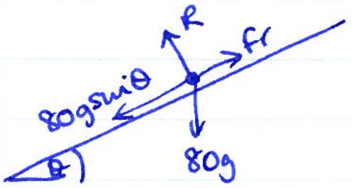
due to friction.

Similarly due to weight $\quad P = 80g\sin 35 \times 1.5 = 674.53$ Watts

$\therefore$ Total power required = 1059.86 Watts

$\therefore P = 1060$ Watts (4 SF)

(v) If initial speed = 0.5 ms^{-1} at top of ramp. Does it reach 3 ms^{-1}?



change in potential energy = change in kinetic e.

$\therefore$ mgh - wd by friction = $\frac{1}{2}m(v^2 - u^2)$

$\therefore \; 980 - \dfrac{392}{\tan\theta} = 40(v^2 - 0.5^2)$

$\therefore \; v = 3.279....$

So the speed reaches 3 ms^{-1} whilst on the ramp.

Q3.

part	mass (g)	c of g. x, y, z
blade	250	0, 4, 0
handle	125	0, 20, 9
total	375	

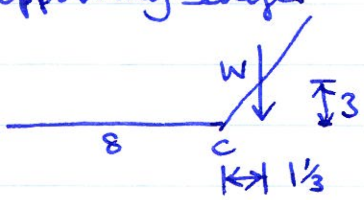
$$M\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \sum m_i\begin{pmatrix}x_i\\ y_i\\ z_i\end{pmatrix}$$

$$\therefore \; 375\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 250\begin{pmatrix}0\\ 4\\ 0\end{pmatrix} + 125\begin{pmatrix}0\\ 20\\ 9\end{pmatrix}$$

$$\therefore \; \begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}0\\ 9\tfrac{1}{3}\\ 3\end{pmatrix}$$

$\therefore$ Centre of mass $(\bar{x}, \bar{y}, \bar{z})$ at $(0, 9\tfrac{1}{3}, 3)$

(ii) Will topple over if the centre of mass is over or beyond the supporting edge.



Take moments about edge CD then

CW = $W \times 1\tfrac{1}{3}$

ACW = 0

So will topple over.

(iii) If $\bar{y} = 14$ and $\bar{z} = 6$?

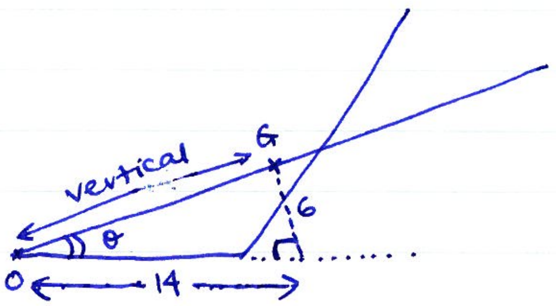
part	mass	cof g (x, y, z)
blade	250	0, 4, 0
handle	125	0, 20, 9
rod	125	0, 28, 15
total	500	

$$\therefore 500\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 250\begin{pmatrix}0\\4\\0\end{pmatrix} + 125\begin{pmatrix}0\\20\\9\end{pmatrix} + 125\begin{pmatrix}0\\28\\15\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}0\\ \frac{7000}{500}\\ \frac{3000}{500}\end{pmatrix} = \begin{pmatrix}0\\14\\6\end{pmatrix}$$

$\therefore$ centre of mass $(\bar{x}, \bar{y}, \bar{z}) = (0, 14, 6)$

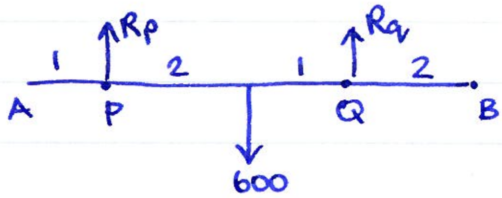
(iv)



$\tan\theta = \frac{6}{14}$

$\therefore \theta = \tan^{-1}\left(\frac{6}{14}\right) = 23.2°$ (3sf)

4(a) (i)



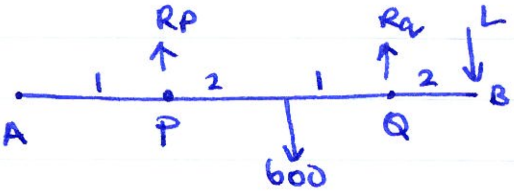
$R_p + R_q = 600$

moments about P $\quad R_q \times 3 = 600 \times 2$

$\therefore R_q = 400$ N

$\therefore R_p = 200$ N

(ii) On the point of tipping when moments about Q



$3R_p + 2L = 600 \times 1 \rightarrow$ so $L = 300$

OR moments about P $\quad$ [If about to tip then $R_p = 0$]

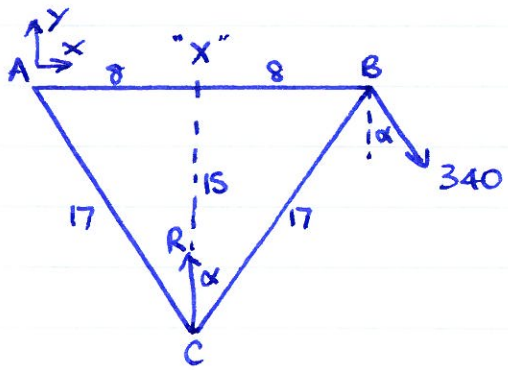
$600 \times 2 + 5L = 3R_q \qquad$ so $R_q = L + 600$

So $\quad 5L = 3(L+600)$

$\therefore 2L = 3\times 600 - 2\times 600$

$\therefore L = 300$ N.

(b)



(i) Moments about A

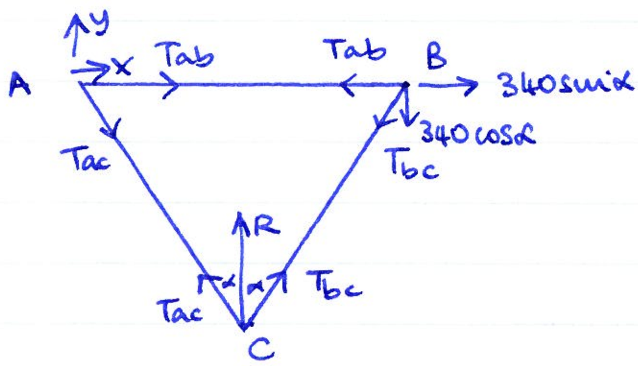
$8R = 340\cos\alpha \times 16$

but $\cos\alpha = \frac{15}{17}$

$\therefore R = \frac{20\times 15}{8} \times 16$

$\therefore R = 600$ N.

(ii)



$\sin\alpha = \frac{8}{17}$

(iii) At point C $\quad T_{ac}\cos\alpha + T_{bc}\cos\alpha + 600 = 0$ (1) (vertically)

At point B $\quad T_{ab} + T_{bc}\sin\alpha = 340\sin\alpha$ (2) (horizontally)

$T_{bc}\cos\alpha + 340\cos\alpha = 0$ (3)

$\therefore T_{bc} = -340$ N (compression)

Subs in (2) $\quad \therefore T_{ab} = 340\sin\alpha + 340\sin\alpha$

$\therefore T_{ab} = 320$ N

subs in (1) $\quad T_{ac}\cos\alpha - 340\cos\alpha + 600 = 0$

$\therefore T_{ac} \times \frac{15}{17} = 340 \times \frac{15}{17} - 600$

$\therefore T_{ac} = -340$ N (compression)