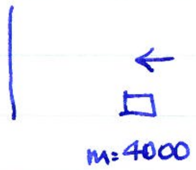
M2 January 2008 Mrs K Wright

1.(a)



$t = 8$ seconds

$F = 1500$ N

(i) hits the wall $F = ma$

$$1500 = 4000a$$

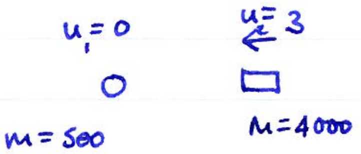
$$\therefore a = \frac{15}{40} = \frac{3}{8} \text{ ms}^{-2}$$

using $v = u + at$

$$v = 0 + \frac{3}{8} \times 8$$

$$\therefore v = 3 \text{ ms}^{-1}$$

(ii)



$e = 0.2$

conservation of linear momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$\therefore 500(0) + 4000(3) = 500\,v_1 + 4000\,v_2$$

$$\therefore v_1 + 8v_2 = 24 \quad (1)$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \therefore (u_1 - u_2)\,0.2 = v_2 - v_1$$

$$\therefore (0 - 3)\,0.2 = v_2 - v_1$$

$$\therefore v_1 - v_2 = 0.6 \quad (2)$$

from (1) and (2) $9v_2 = 23.4$

$\therefore v_2 = 2.6$ m/s velocity of beam

$\therefore v_1 = 3.2$ m/s Velocity of stone.

(iii) energy lost in the impact

for beam $= \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 4000 \times (2.6^2 - 3^2) = -4480$ J

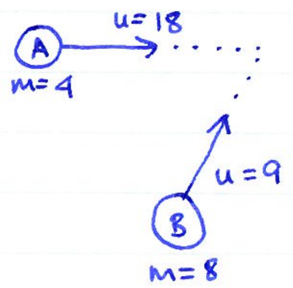
for stone $= \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 500 \times (3.2^2 - 0) = 2560$ J

change in energy $= 2560 - 4480 = -1920$

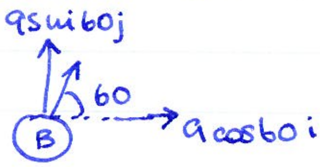
$\therefore$ loss of 1920 Joules.

1(b)

(i)



momentum = mv



velocity of B

momentum for A $= 4 \times 18i = 72i$ Ns

momentum for B $= 8(9\cos 60\, i + 9\sin 60\, j)$

$= 72 \times \frac{1}{2} i + 72 \times \frac{\sqrt{3}}{2} j$

$= 36i + 36\sqrt{3}\, j$

(ii) coalesce after impact, common velocity $(ui + vj)$ m/s

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$4(18i) + (36i + 36\sqrt{3}\, j) = 4v_1 + 8v_1$

$\therefore\ 72i + 36i + 36\sqrt{3}\, j = 12(ui + vj)$

$\therefore\ 72i + 36i + 36\sqrt{3}\, j = 12ui + 12vj$

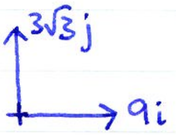
comparing i : $108 = 12u$

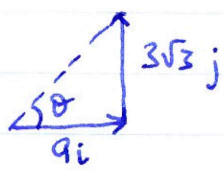
$\therefore\ u = 9$ m/s

comparing j : $36\sqrt{3} = 12v$

$\therefore\ v = 3\sqrt{3}$ m/s.

(iii) after

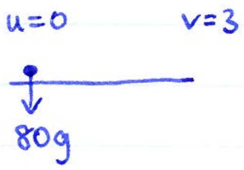




$\tan\theta = \frac{3\sqrt{3}}{9} = \frac{\sqrt{3}}{3}$ $\quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

Q2 (i)

(A)



kinetic energy gain $= \frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2} \times 80 \times (9 - 0)$

$= 360$ Joules

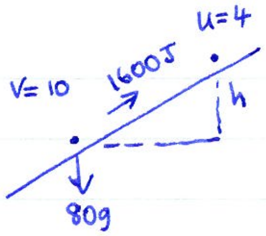
(B) If distance = 12 metres

Average driving force: workdone = ke

$\therefore\ F \times 12 = 360$

$\therefore\ F = 30$ N.

2(ii)



change in ke $= \frac{1}{2}m(10^2-4^2) = \frac{1}{2}\times 80 \times (100-16)$

$= 3360$ J

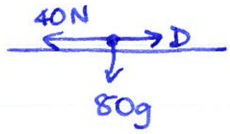
work done $= mgh - 1600$

$\therefore\ 80gh - 1600 = 3360$

$\therefore\ h = 6.3265....$

$\therefore\ h = 6.33$ metres (3sf)

(iii) (A)



Power $= 200$ watt

$P = Fv$

$\therefore\ 200 = 40v$

$\therefore\ v = 5$ m/s

(B) If $v = 0.5$ m/s and $a = 2$ m/s^2

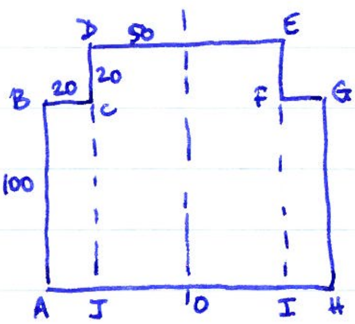
To calculate Power, need new force, so using $F=ma$, with resistance of 40N

$D - 40 = 80 \times 2$

$\therefore\ D = 200$ N

$\therefore$ Power $= Dv = 200 \times 0.5 = 100$ watt.

3.(i)



part	mass	y	z
rectangle	120×140	0	60
L square	−20×20	−60	110
R square	−20×20	60	110
	16000	$\bar{y}$	$\bar{z}$

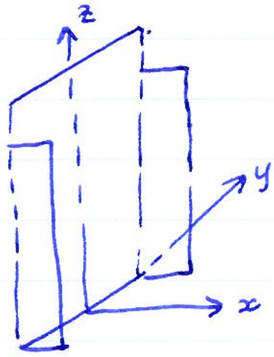
$\therefore\ M\begin{pmatrix}\bar{y}\\ \bar{z}\end{pmatrix} = \sum m_i \begin{pmatrix}y_i\\ z_i\end{pmatrix}$

$\therefore\ 16000\begin{pmatrix}\bar{y}\\ \bar{z}\end{pmatrix} = 16800\begin{pmatrix}0\\ 60\end{pmatrix} - 400\begin{pmatrix}-60\\ 110\end{pmatrix} - 400\begin{pmatrix}60\\ 110\end{pmatrix} = \begin{pmatrix}0\\ 920000\end{pmatrix}$

$\therefore\ \begin{pmatrix}\bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}0\\ 57.5\end{pmatrix}$

$\therefore$ centre of mass $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 57.5)$

(ii)



part	mass	x	y	z
end 1	20×100	10	−50	50
end 2	20×100	10	50	50
side	120×100	0	0	60

$$M\bar{x} = \Sigma m_i x_i$$

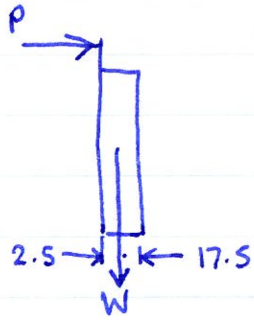
$$16000\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 2000\begin{pmatrix}10\\ -50\\ 50\end{pmatrix} + 2000\begin{pmatrix}10\\ 50\\ 50\end{pmatrix} + 12000\begin{pmatrix}0\\ 0\\ 60\end{pmatrix}$$

$$\therefore\ 16000\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}40000\\ 0\\ 920000\end{pmatrix}$$

$$\therefore\ \begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}2.5\\ 0\\ 57.5\end{pmatrix}$$

$\therefore$ centre of mass $(\bar{x}, \bar{y}, \bar{z}) = (2.5, 0, 57.5)$

(iii)



If on the point of tipping over A#

$P \times 120 = W \times 17.5$ $\qquad W = 72$

$\therefore\ P = \dfrac{72 \times 17.5}{120} = 10.5\ \text{N}$

(iv) Coefficient of friction μ, using limiting friction $F \leq \mu R$

$R = 72$

$\therefore\ F = \mu \times 72$

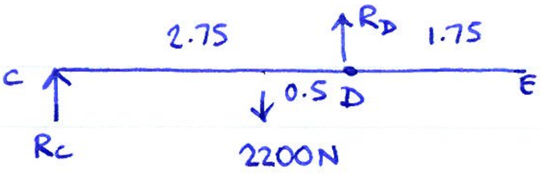
If slides before tips $\quad F < 10.5$

$\therefore\ 72\mu < 10.5$

$\therefore\ \mu < \dfrac{10.5}{72}$ ie $\mu < 0.1458\dot{3}$

$\therefore\ \mu < 0.146$ (3sf)

4.



$\frac{4.5}{2} = 2.25$

$2.75 - 2.25 = 0.5$ metres

(i) anticlockwise moment of weight about D

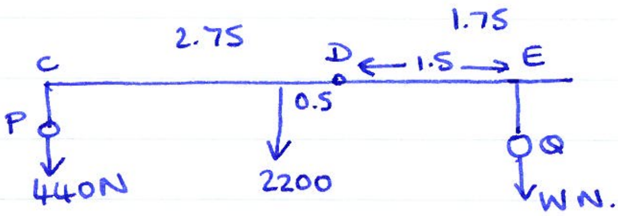
$= 2200 \times 0.5$

$= 1100$ N.

$\therefore \quad R_C \times 2.75 = 2200 \times 0.5$

$\therefore R_C = 400$ N.

(ii)

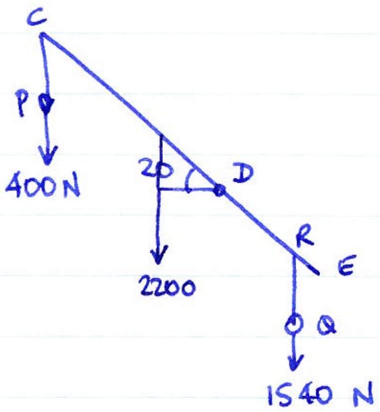


In equilibrium, taking moments about D

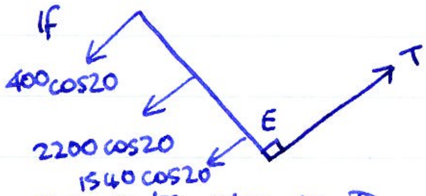
$\therefore \quad 440 \times 2.75 + 2200 \times 0.5 = W \times 1.5$

$\therefore \quad W = 1540$ N

(iii)



(A) If



moments about D

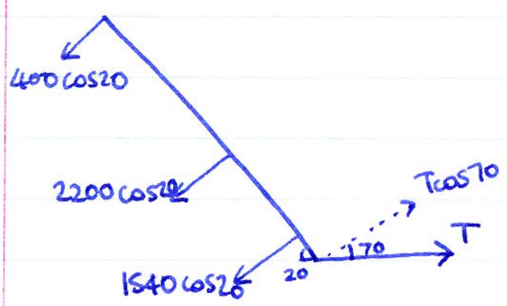
$400\cos 20 \times 2.75 + 2200\cos 20 \times 0.5$

$= 1540\cos 20 \times 1.5 - T \times 1.75$

$\therefore \quad T = 59.066\ldots$

$\therefore \quad T = 59.1$ N (3sf)

(B)



moments about D

$400\cos 20 \times 2.75 + 2200\cos 20 \times 0.5$

$= 1540\cos 20 \times 1.5 - T\cos 70 \times 1.75$

$\therefore \quad T = 172.698\ldots$

$\therefore \quad T = 173$ N (3sf)