M2 January 2007. Mrs K Wright

1. $\overrightarrow{u=10}$ $u=0$

0.5 kg 29.5 kg

$\rightarrow v_2$ $\rightarrow v_1$

$e = 0.8$

$e = \dfrac{v_1 - v_2}{10 - 0} = 0.8$ $\therefore v_1 - v_2 = 8$ ①

using conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore 0.5(10) + 29.5(0) = 0.5\,v_2 + 29.5\,v_1$

$\therefore v_2 + 59v_1 = 10$ ②

using ① and ②

$60v_1 = 18$

$\therefore v_1 = 0.3$ m/s

$\therefore v_2 = -7.7$ m/s

the ball changes direction after impact, opposite to initial motion.

(ii) (A) $u = 10 \rightarrow$ $u=0$ $\rightarrow v$

0.5 kg 29.5 kg 30 kg coalesce after impact

$0.5(10) + 29.5(0) = 30(v)$

$\therefore v = \frac{1}{6}$ m/s.

(B) throwing the snow off the sledge does not change the speed the sledge travels at as no external forces are applied.

(iii) $u = 2 \rightarrow$ $\overrightarrow{v+10}$ $\rightarrow v$

40 kg 0.5 kg 39.5 kg

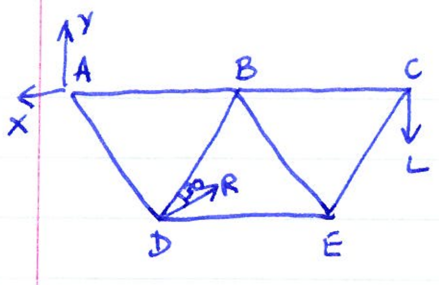
before — after —

$\therefore 40(2) = 39.5(v) + 0.5(v+10)$

$\therefore 80 = 40v + 5$

$\therefore v = \frac{75}{40} = 1.875$ m/s.

Q2.



plane at D smooth so R but no Friction

let the length of each rod be $2l$

(i) Horizontal equilibrium $\quad R\sin 60 = X \quad$ (or $R\cos 30 = X$)

$$\therefore R = \frac{2X}{\sqrt{3}}$$

Vertical Equilibrium $\quad Y + R\cos 60 = L$

$$\therefore Y + \tfrac{1}{2}R = L$$

(ii) Taking moments about A excludes X and Y

$$\therefore R \times 2l = L \times 4l$$

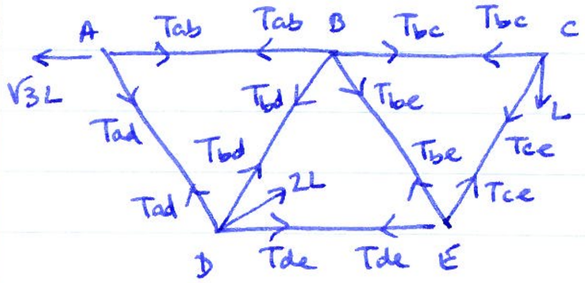
$$\therefore R = 2L$$

$$\therefore X = R\frac{\sqrt{3}}{2} = 2L\frac{\sqrt{3}}{2} = \sqrt{3}L \qquad \therefore X = \sqrt{3}L$$

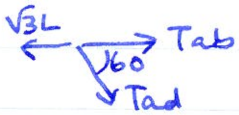
and $\quad Y + \tfrac{1}{2} \times 2L = L$

$$\therefore Y = 0$$

(iii)



(iv) Consider A

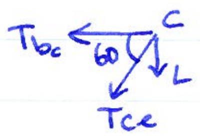


Resolving vertically $\quad T_{ad}\cos 30 = 0$

$$\therefore T_{ad} = 0$$

(v) Resolving horizontally at A $\quad T_{ab} = \sqrt{3}L$ (tension) since $T_{ad}\cos 60 = 0$

Resolving vertically at C $\quad T_{ce}\cos 30 + L = 0$



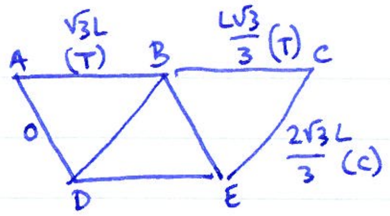
$$\therefore T_{ce} = -\frac{L}{\cos 30} = -\frac{2L}{\sqrt{3}} = -\frac{2L\sqrt{3}}{3}$$

(so in compression)

Resolving horizontally at C $\quad T_{bc} + T_{ce}\cos 60 = 0$

$$\therefore T_{bc} - \frac{2L}{\sqrt{3}} \times \frac{1}{2} = 0 \qquad \therefore T_{bc} = \frac{L\sqrt{3}}{3} \text{ (tension)}$$

(vi)



If we resolve at B

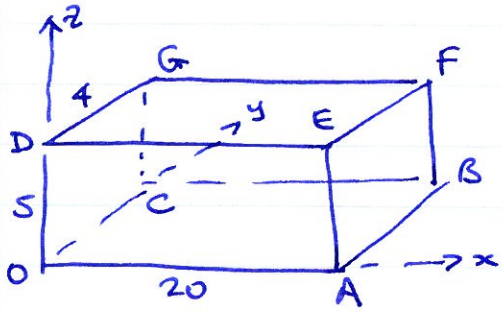
$\leftrightarrow$ $T_{ab} + T_{bd}\cos 60 = T_{bc} + T_{be}\cos 60$

$\updownarrow$ $T_{bd}\cos 30 + T_{be}\cos 30 = 0$

$\therefore T_{bd} = -T_{be}$

so T_{bd} and T_{be} have equal magnitude and opposite signs - one in tension, one in compression.

Q3.



(i) Coordinates of mass of open box with no top or bottom, will be at centre of x, centre of y and centre of z

$= (10, 2, 2.5)$

(ii) With base added to the 4 sides.

part	mass	x	y	z
original	100+100+20+20	10	2	2.5
bottom	80	10	2	0
	320	$\bar{x}$	$\bar{y}$	$\bar{z}$

Since $\bar{x}$ and $\bar{y}$ are same for sides and base we really need to only calculate the $\bar{z}$

$$320\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 240\begin{pmatrix}10\\ 2\\ 2.5\end{pmatrix} + 80\begin{pmatrix}10\\ 2\\ 0\end{pmatrix}$$

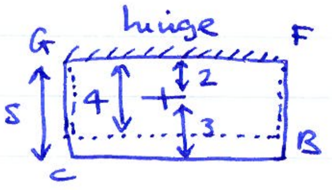
$$\therefore 320\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}3200\\ 640\\ 600\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}10\\ 2\\ 1.875\end{pmatrix}$$

$\therefore$ centre of mass $(\bar{x}, \bar{y}, \bar{z})$ is $(10, 2, 1.875)$

(iii) Top face now added and hangs down next to plane FGCB.

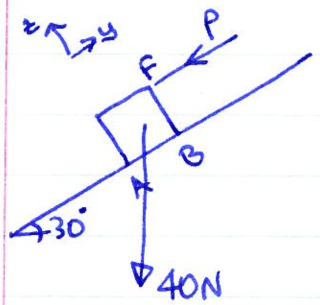
part	mass	x	y	z
base + box	320	10	2	1.875
top on side	80	10	4	3
	400.			

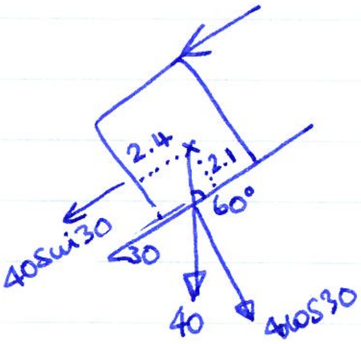


$$\therefore\ 400\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 320\begin{pmatrix}10\\ 2\\ 1.875\end{pmatrix} + 80\begin{pmatrix}10\\ 4\\ 3\end{pmatrix} = \begin{pmatrix}4000\\ 960\\ 840\end{pmatrix}$$

$$\therefore\ \begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \begin{pmatrix}10\\ 2.4\\ 2.1\end{pmatrix}$$

(iv) Now box is on a sloping plane. Box does not slip and is on the point of tipping about edge AO.





clockwise moment = Fd

where F = 40 cos 30 due to weight

d = 2.4 (cm)

anticlockwise moment = Fd

where F = 40 sin 30 due to weight

d = 2.1 (cm)

$\therefore$ Total overall clockwise moment (due only to weight) $= 40\cos 30 \times 0.024 - 40\sin 30 \times 0.021$

$= 0.41138...$

$= 0.411$ Nm (3sf)

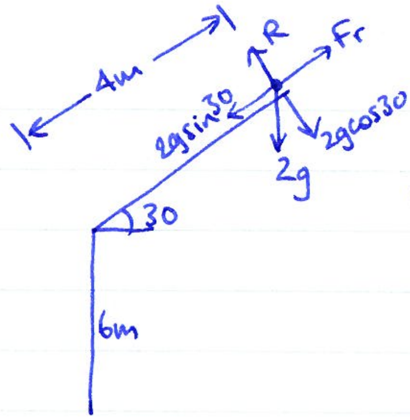
(v) Now taking moments about edge OA

$$40\cos 30 \times 0.024 - 40\sin 30 \times 0.021 = P \times \frac{5}{100} \quad \text{(all in m)}$$

$\therefore\ P = 0.41138... \div 0.05$

$\therefore\ P = 8.227$

$\therefore\ P = 8.23$ N (3sf).

Q4. 

$\mu = 0.75$

(i) limiting friction $F \leq \mu R$

$R = 2g\cos 30$

$F_r = 2g\sin 30$ if at rest about to slide but we do not know if this is the case so

use $F_r = \mu R = 0.75 \times 2g\cos 30$

$\therefore F_r = 12.730\ldots$ N

Limiting frictional force : $F_r = 12.7$ N (3sf)

Will the tile slide ? Look at $F_r = 2g\sin 30 = 2g \times \frac{1}{2} = 9.8$ N

Since 9.8 N < 12.7 N the tile will not slide.

(ii) tiles raised up 6m from the ground, and the only work done is against gravity, and then slid up to A.

(A) gravitational potential energy = mgh.

$h_1 = 4\sin 30 = 2$

$\therefore mgh = 2 \times 9.8 \times (6+2) = 156.8$ Joules.

(B) work done against friction per tile $= F_r d$

$= 12.730\ldots \times 4$

$= 50.9$ Joules

(C) Average power required to raise 10 tiles per minute to point A from the ground

$$\text{Power} = Fv = F\frac{d}{t} = \frac{\text{work done}}{\text{time}}$$

10 tiles per minute is approximately 1 tile for each 6 seconds.

Total work done = wd against gravity + work done against Fr

$= 156.8 + 50.9$

$= 207.7$

$\therefore \text{Power} = \frac{207.7}{6} = 34.6$ Watts.

(iii) tile kicked off the roof. When at x metres from the edge of the roof $u = 4$ m/s then hits the ground at 9m/s.
Work done against sliding and other resistances = 90 Joules.
To find x:

work done = change in kinetic energy

$mgh - 90 = \frac{1}{2}m(v^2 - u^2)$

$\therefore \quad 2 \times 9.8 \times (6 + x \sin 30) = \frac{1}{2} \times 2 \times (81 - 16) + 90$

$\therefore \quad 19.6(6 + x \sin 30) = 65 + 90$

$\therefore \quad x = 3.816$ metres

$\therefore x = 3.82$ m (3sf).

Thinking about what happens with NLM2

$mg\sin 30 - Fr = ma$ for motion on the roof

So using energy methods we can use work done = force × distance and

from suvat $F = ma$ $\qquad a = \frac{1}{2}(v^2 - u^2)$