M1. January 2010

1. $\downarrow$ $u = 4.2$ m/s
$t = 1.5$ seconds
$a = 9.8$ m/s^2

(i) $v = u + at$
$\therefore v = 4.2 + 9.8 \times 1.5$
$\therefore v = 18.9$ m/s

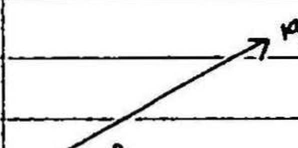
(ii) $v^2 = u^2 + 2as$ or $s = ut + \frac{1}{2}at^2$

$18.9^2 = 4.2^2 + 19.6s$
$\therefore s = 17.325$ metres
$\therefore s = 17.3$ m (3sf)

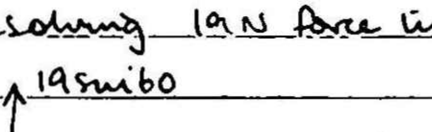
$\therefore s = 4.2 \times 1.5 + 4.9 \times 1.5^2$
$\therefore s = 17.325$
$\therefore s = 17.3$ m (3sf)
(preferable as uses given info)

(iii) 5m above the ground $s = 17.325 - 5 = 12.325$ metres
$v^2 = u^2 + 2as$
$\therefore v^2 = 4.2^2 + 19.6 \times 12.325$
$\therefore v = 16.1$ m/s

2. (i)

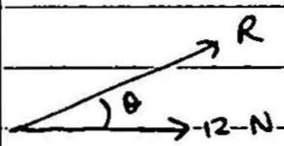


Resolving 19N force into components



$\therefore \text{Resultant}^2 = (12 + 19\cos 60)^2 + (19\sin 60)^2$
$\therefore R^2 = 733$
$\therefore R = 27.1$ N (3sf)

(ii)



$\tan\theta = \dfrac{19\sin 60}{12 + 19\cos 60} = 0.7653\ldots$
$\therefore \theta = 87.4°$ (3sf)

3.

	$u = 9 \rightarrow$	$u = 2 \rightarrow$	$u = 2.75 \rightarrow$
before	(P) m	(Q) 0.8kg	(R) 0.4kg
after	$\leftarrow$ $v = 3.5$	$\rightarrow$ $v = 3.5$	

(i) a) using conservation of momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$m(9) + 0.8(2) = m(-3.5) + 0.8(3.5)$$

$$\therefore 12.5m = 1.2$$

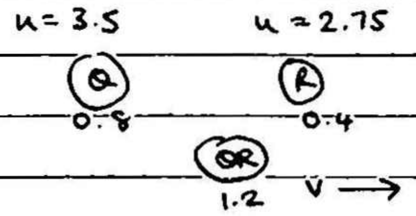
$$\therefore m = \frac{1.2}{12.5} = 0.096 \text{ kg}$$

b) change in momentum $= mv - mu = m(v-u)$

$$= 0.096(-3.5-9)$$

$$= -1.2 \text{ Ns.}$$

(ii) Q and R

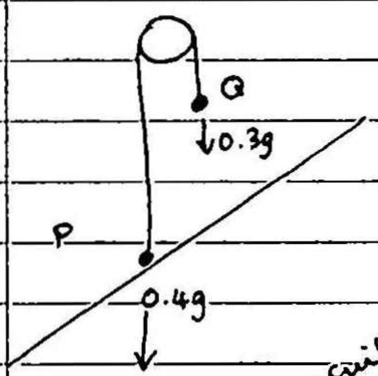


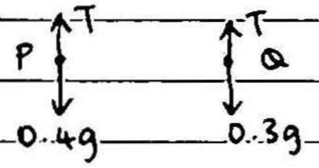
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ so $3.5 \times 0.8 + 2.75 \times 0.4 = 1.2V$

$$\therefore V = 3.25 \text{ m/s}$$

4.

(i)

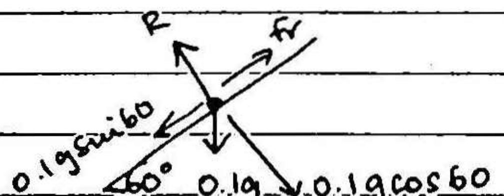




So at rest

$\therefore T = 0.3g$ at Q

so at P $0.4g - T = 0.1g$



limiting equilibrium $F_r = \mu R$

(a) $\therefore R = 0.1g\cos 60$ (perpendicular component)

$Fr = 0.1g\sin 60$ (parallel component)

(b) If $F_r \le \mu R$ then $\mu = \frac{Fr}{R} = \frac{0.1g\sin 60}{0.1g\cos 60} = \tan 60 = \sqrt{3}$

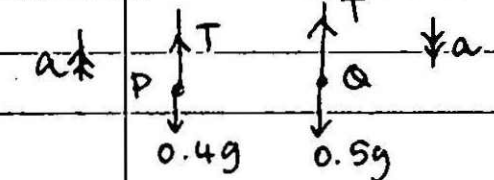
$\therefore \mu = \sqrt{3} = 1.73$ (3sf)

(ii) Now Q increased from 0.3g to 0.5g and released from rest

$\therefore$ at P $T - 0.4g = 0.4a$

at Q $0.5g - T = 0.5a$

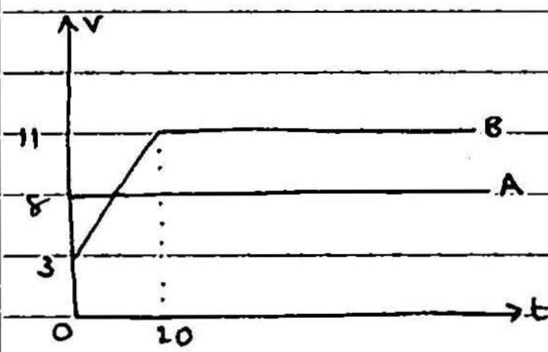
(adding) $0.1g = 0.9a$



$\therefore \quad a = \dfrac{0.1g}{0.9} = 1.09 \ \ m/s^2 \quad (3sf)$

and (subtracting) $\quad T = 4.36 \ N. \quad (3sf)$

5.



(i) When A and B have the same speed (the point of intersection on the v-t graph)

for A : $v = 8$

for B : $v = u + at$

$8 = 3 + \left(\dfrac{11-3}{20}\right)t$

$\therefore \ 5 = \dfrac{8}{20} t$

$\therefore \ t = 12.5$ seconds

[the gradient $\Rightarrow$ "$\dfrac{\Delta y}{\Delta x}$"]

$\therefore \quad a = \dfrac{11-3}{20} = \dfrac{8}{20}$

(b) When B overtakes A, they have the same displacement so the area underneath each graph will be equal

for A : $\quad s = 8 \times T$

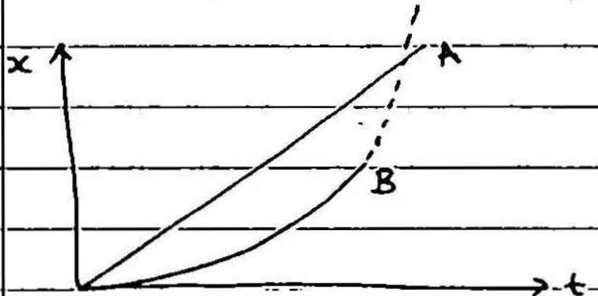
for B : $\quad s = \frac{1}{2}(11+3) \times 20 + 11(T-20)$

$\therefore \quad 8T = 140 + 11T - 220$

$\therefore \quad 3T = 80$

$\therefore \ T = \dfrac{80}{3} = 26.7$ seconds.

(iii) the displacement time graphs for A and B from $t = 0$ to 26.7



A : $s = ut$ so constant and the gradient is the velocity

B : as acceleration $= 0.04$ the graph increases from 0, then when acceleration $= 0$ the gradient is the velocity

6. $\quad v = 0.006t^2 - 0.18t + k \quad$ time to swim one end to other $= 28.4$ secs.

(i) acceleration $= \dfrac{dv}{dt} = 0.012t - 0.18$

(ii) If the minimum speed is 0.65 m/s

For min or max speed $\frac{dv}{dt} = 0 \quad \therefore \quad 0.012t - 0.18 = 0$

$\therefore \quad t = \frac{0.18}{0.012} = 15$ seconds

When $t = 15$

$v = 0.006(225) - 0.18(15) + k = 0.65$

$\therefore k = 2$

(iii) If $v = 0.006t^2 - 0.18t + 2$ and $v = \frac{dx}{dt}$

$\therefore x = 0.006\frac{t^3}{3} - 0.18\frac{t^2}{2} + 2t + c$

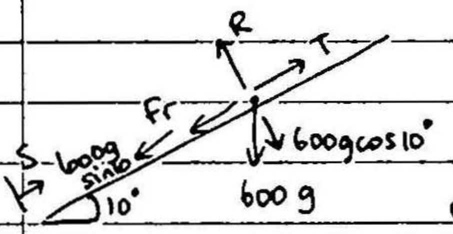
When $x = 0$ at $t = 0$ then $c = 0$ so $x = 0.002t^3 - 0.09t^2 + 2t$

Since the length of the pool is swum in 28.4 seconds, put $t = 28.4$

$\therefore x = 0.002(28.4)^3 - 0.09(28.4)^2 + 2(28.4)$

$\therefore x = 30.0$ metres (3sf)

7.



$\mu = 0.15$

$a = 0.11 \text{ m/s}^2$

(i) $T - Fr - 600g\sin 10 = 600 \times 0.11$ (F=ma) up

Since $R = 600g\cos 10$

and $F = \mu R$ then $f = 0.15 \times 600g\cos 10$

$\therefore T = 1955.65$

$\therefore T = 1960$ N (3sf)

(ii) Cable breaks after going 10 metres up the slope – so no longer any T

(a) $\therefore -F - 600g\sin 10 = 600a$

$\therefore -0.15 \times 600g\cos 10 - 600g\sin 10 = 600a$

$\therefore a = -3.15 \text{ m/s}^2$ (3sf)

so deceleration 3.15 m/s^2 up the slope

(b) the motion continues until comes to rest, then goes back down.

we need to find the speed at the time of the cable break.

using $v^2 = u^2 + 2as$ then $v^2 = 0^2 + 2 \times 0.11 \times 10 = 2.2$

$v = \sqrt{2.2}$

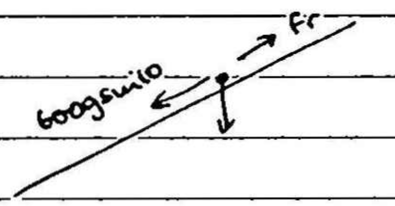
So $u = \sqrt{2.2}$, $v = 0$, $a = -3.15$ then for distance up the slope

$0 = 2.2 + 2(-3.15)s$

$s = \frac{22}{63}$, so the total distance up the slope is $10 + \frac{22}{63}$ metres.

Time to go up $v = u + at$ $\therefore t = \frac{0 - \sqrt{2.2}}{-3.15} = 0.471$ seconds

Time to come down - we need to change the acceleration to gravity with friction



$600g\sin 10 - Fr = 600a$ $\quad Fr = 0.15 \times 600g\cos 10$

$600g\sin 10 - 0.15 \times 600g\cos 10 = 600a$

$\therefore a = 0.254 \ m/s^2$

So $s = ut + \frac{1}{2}at^2$ $\quad u = 0,\ a = 0.254,\ s = 10\frac{22}{63}$

$10\frac{22}{63} = 0 + \frac{1}{2} \times 0.254 \times t^2$

$\therefore t = 9.025$ seconds.

So total time to return back to original position $= 9.025 + 0.471$

$= 9.50$ seconds. (3sf).