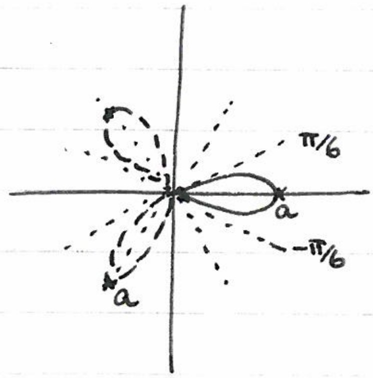
FP2 January 2006. Polar Coordinates.

(a) $r = a\cos 3\theta \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(i)

					$\cos(\frac{\pi}{2})$	$\cos(\pi)$	
θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos 3\theta$	0	-1	0	1	0	-1	0

$\frac{1}{\sqrt{2}}$ $\left(\frac{\pi}{12}\right)$ $(\cos(\frac{\pi}{4}))$ $\quad -\frac{1}{\sqrt{2}}$ $\left(\frac{\pi}{4}\right)$ $\cos(\frac{3\pi}{4})$ $\quad -\frac{1}{\sqrt{2}}$ $\left(\frac{5\pi}{12}\right)$



(ii) area of sector $= \int \frac{1}{2} r^2 d\theta$

1st loop between $\theta = -\frac{\pi}{6}$ and $\theta = +\frac{\pi}{6}$

area of 1st loop is $\int_{-\pi/6}^{\pi/6} \frac{1}{2}(a\cos 3\theta)^2 d\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 6\theta = 2\cos^2 3\theta - 1$

$\cos^2 3\theta = \frac{1}{2}(\cos 6\theta + 1)$

$\frac{d}{dx}(\sin 6\theta) = 6\cos 6\theta$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{4}a^2(\cos 6\theta + 1)\, d\theta$

$= \frac{1}{4}a^2\left[\frac{1}{6}\sin 6\theta + \theta\right]_{-\pi/6}^{\pi/6}$

$= \frac{1}{4}a^2\left[\frac{1}{6}\sin(\pi) + \frac{\pi}{6} - \frac{1}{6}\sin(-\pi) + \frac{\pi}{6}\right]$

$= \frac{\pi a^2}{12}$

(b) $\int_0^{3/4} \frac{1}{\sqrt{3-4x^2}}\, dx$

$y = \arcsin(x)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\frac{1}{\sqrt{3-4x^2}} = \frac{1}{\sqrt{4(\frac{3}{4}-x^2)}}$

$\frac{1}{\sqrt{3-4x^2}} = \frac{1}{\sqrt{4(\frac{3}{4}-x^2)}} = \frac{1}{2\sqrt{\frac{3}{4}-x^2}}$

$\int_0^{3/4} \frac{1}{\sqrt{3-4x^2}}\, dx = \left[\frac{1}{2}\arcsin\left(\frac{x}{\sqrt{3/4}}\right)\right]_0^{3/4}$

$\frac{1}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right)$

$= \frac{1}{2}\arcsin\sqrt{\frac{3}{4}} - \frac{1}{2}\arcsin 0$

$= \frac{1}{2}\left(\frac{\pi}{3}\right) - 0$

$= \frac{\pi}{6}$

FP2 Jan 2006 contd.

1 (c)

$$\int_0^1 \frac{1}{(1+3x^2)^{3/2}}\,dx$$

$1+3x^2 \equiv 1+\tan^2\theta$

if $\tan\theta = \sqrt{3}x$

$x = \frac{1}{\sqrt{3}}\tan\theta$

$\frac{dx}{d\theta} = \frac{1}{\sqrt{3}}\sec^2\theta$

when $x=1$ $\tan\theta=\sqrt{3}$

$\therefore \theta = \frac{\pi}{3}$

when $x=0$ $\tan\theta = 0$

$\theta = 0$

$$= \int_0^{\pi/3} \frac{1}{(\sec^2\theta)^{3/2}} \times \frac{1}{\sqrt{3}}\sec^2\theta\,d\theta$$

$$= \int_0^{\pi/3} \frac{1}{\sqrt{3}\sec^3\theta} \times \sec^2\theta\,d\theta$$

$$= \int_0^{\pi/3} \frac{d\theta}{\sqrt{3}\sec\theta}$$

$$= \int_0^{\pi/3} \frac{1}{\sqrt{3}}\cos\theta\,d\theta$$

$$= \frac{1}{\sqrt{3}}\Big[\sin\theta\Big]_0^{\pi/3}$$

$$= \frac{1}{\sqrt{3}}\left(\sin(\pi/3) - \sin(0)\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2} - 0\right)$$

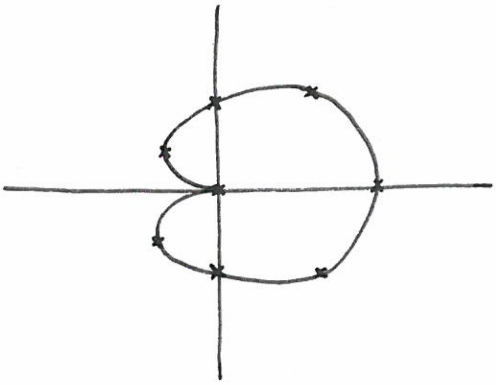
$$= \frac{1}{2}$$

FP2 June 2006.

1(a) $r = a(\sqrt{2} + 2\cos\theta)$ $\quad -\frac{3}{4}\pi \le \theta \le \frac{3\pi}{4}$ $\quad a > 0.$

(i)

θ	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\frac{r}{a}$	0	$\sqrt{2}$	$2\sqrt{2}$	π	$2\sqrt{2}$	$\sqrt{2}$	



(ii) region enclosed by curve $= \displaystyle\int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2} r^2 \, d\theta = \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2} a^2 (\sqrt{2} + 2\cos\theta)^2 \, d\theta$

$$= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2} a^2 (\sqrt{2})^2 (1 + \sqrt{2}\cos\theta)^2 \, d\theta$$

$$= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} a^2 (1 + 2\sqrt{2}\cos\theta + 2\cos^2\theta) \, d\theta$$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = \cos 2\theta + 1$

$$= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} a^2 (1 + 2\sqrt{2}\cos\theta + \cos 2\theta + 1) \, d\theta$$

$\frac{d}{d\theta}(\sin 2\theta) = 2\cos 2\theta$

$$= a^2 \left[2\theta + 2\sqrt{2}\sin\theta + \frac{1}{2}\sin 2\theta \right]_{-3\pi/4}^{3\pi/4}$$

$$= a^2 \left(\frac{3\pi}{2} + 2 + \left(-\frac{1}{2}\right) \right) - \left(-\frac{3\pi}{2} - 2 + \frac{1}{2} \right)$$

$$= a^2 (3\pi + 3)$$

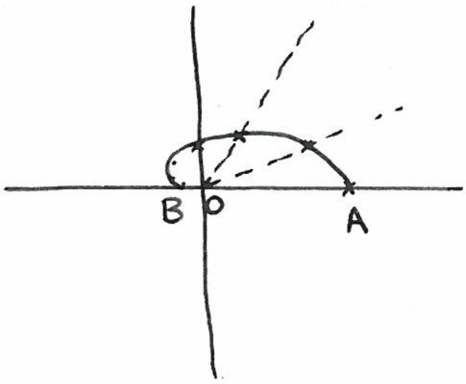
$$= 3a^2(\pi + 1)$$

FP2 January 2007

1(a) $r = ae^{-k\theta}$ $\quad 0 \le \theta \le \pi \quad a > 0 \quad k > 0 \quad A\ (\theta = 0) \quad B\ (\theta = \pi)$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	1	0.59	0.35	0.21	0.12	0.07	0.04

$a = k = 1$



(ii) Area of curve $= \int_0^\pi \frac{1}{2} r^2 \, d\theta = \int_0^\pi \frac{1}{2}\left(ae^{-k\theta}\right)^2 d\theta$

$$= \int_0^\pi \frac{1}{2} a^2 e^{-2k\theta} \, d\theta$$

$$= \left[\frac{1}{2}\left(\frac{1}{-2k}\right) a^2 e^{-2k\theta} \right]_0^\pi$$

$$= \left(-\frac{1}{4k} a^2 e^{-2\pi k}\right) - \left(-\frac{1}{4k} a^2 e^0\right)$$

$$= \frac{a^2}{4k}\left(1 - e^{-2\pi k}\right)$$

(b) $\int_0^{\frac{1}{2}} \frac{1}{3+4x^2} \, dx$ $\qquad$ "$1 + \tan^2\theta \simeq 3 + 4x^2$"

$y = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$

$\frac{dy}{dx} = \frac{1}{x^2 + a^2}$

$$= \int_0^{\frac{1}{2}} \frac{1}{4\left(\frac{3}{4} + x^2\right)} \, dx$$

$$= \frac{1}{4}\left[\frac{1}{\sqrt{3}/4} \arctan\left(\frac{x}{\sqrt{3}/4}\right)\right]_0^{1/2} = \frac{1}{4}\left[\frac{2}{\sqrt{3}} \arctan \frac{2x}{\sqrt{3}}\right]_0^{1/2}$$

$$= \frac{1}{2\sqrt{3}}\left\{\arctan\left(\frac{1}{\sqrt{3}}\right) - \arctan(0)\right\}$$ $\qquad \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

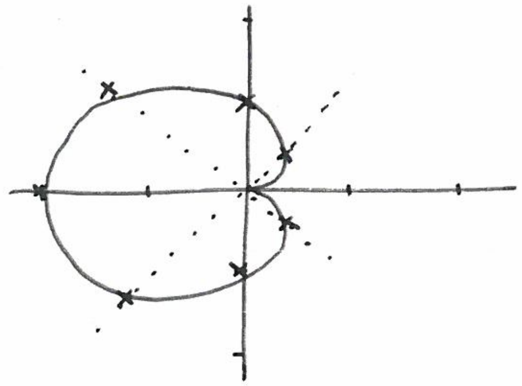
$$= \frac{1}{2\sqrt{3}}\left(\frac{\pi}{6}\right) = \frac{\pi}{12\sqrt{3}}$$

FP2 June 2007.

1(a) $r = a(1-\cos\theta)$

(i)

θ	$-\pi$	$-\frac{3\pi}{4}$	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\frac{r}{a}$	2	1.71	1	0.29	0	0.29	1	1.71	2



(ii) area enclosed by loop $= \int_0^{\pi/2} \frac{1}{2} r^2 \, d\theta$

$$= \int_0^{\pi/2} \frac{1}{2}\left(a^2(1-\cos\theta)^2\right) d\theta$$

$$= \frac{a^2}{2}\int_0^{\pi/2} \left(1+\cos^2\theta - 2\cos\theta\right) d\theta$$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$

$$= \frac{a^2}{2}\int_0^{\pi/2} \left(1 + \frac{1}{2}\cos 2\theta + \frac{1}{2} - 2\cos\theta\right) d\theta$$

$$= \frac{a^2}{2}\left[\frac{3\theta}{2} + \frac{1}{4}\sin 2\theta - 2\sin\theta\right]_0^{\pi/2}$$

$$= \frac{a^2}{2}\left[\frac{3\pi}{4} + 0 - 2\right] - \left[0\right]$$

$$= \frac{a^2}{2}\left(\frac{3\pi}{4} - 2\right)$$

(b) $\int_0^1 \frac{1}{(4-x^2)^{3/2}} \, dx$

since "$-$" use sin or cos

want $4 - x^2 \equiv 4(1-\sin^2\theta)$

or $4(1-\cos^2\theta)$

let $x = 2\sin\theta$ or $2\cos\theta$

$$\int_0^1 \frac{1}{(4-x^2)^{3/2}}\,dx$$

let $x = 2\sin\theta$

$$\frac{dx}{d\theta} = 2\cos\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{(4-4\sin^2\theta)^{3/2}} \times 2\cos\theta\, d\theta$$

$$4-4\sin^2\theta = 4(1-\sin^2\theta)$$
$$= 4\cos^2\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{(4\cos^2\theta)^{3/2}} \times 2\cos\theta\, d\theta$$

$x = 1 \quad \sin\theta = \frac{1}{2} \quad \theta = \frac{\pi}{6}$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{8\cos^3\theta} \times 2\cos\theta\, d\theta$$

$x = 0 \quad \sin\theta = 0 \quad \theta = 0$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{4\cos^2\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{4}\sec^2\theta\, d\theta$$

$$= \frac{1}{4}\Big[\tan\theta\Big]_0^{\pi/6}$$

$$= \frac{1}{4}\left(\tan\frac{\pi}{6} - \tan 0\right)$$

$$= \frac{1}{4} \times \frac{1}{\sqrt{3}}$$

$$= \frac{1}{4\sqrt{3}}$$

FP2. January 2008

1(a) $r = a(1-\cos 2\theta) \qquad 0 \leq \theta \leq \pi$

(i) area enclosed by curve $= \int_0^{\pi} \frac{1}{2} r^2 \, d\theta$

$$= \frac{1}{2}\int_0^{\pi} a^2 (1-\cos 2\theta)^2 \, d\theta$$

$\cos 4\theta = 2\cos^2 2\theta - 1$

$\cos^2 2\theta = \dfrac{\cos 4\theta + 1}{2}$

$$= \frac{1}{2}\int_0^{\pi} a^2 \left(1 + \cos^2 2\theta - 2\cos 2\theta\right) d\theta$$

$$= \frac{1}{2}\int_0^{\pi} a^2 \left(1 + \frac{1}{2}\cos 4\theta + \frac{1}{2} - 2\cos 2\theta\right) d\theta$$

$$= \frac{1}{2} a^2 \left[\frac{3}{2}\theta + \frac{1}{8}\sin 4\theta - \sin 2\theta\right]_0^{\pi}$$

$$= \frac{1}{2}a^2\left\{\left(\frac{3\pi}{2} + 0 - 0\right)\right\} - \left\{\frac{1}{2}a^2\left(3 \times 0 + 0 - 0\right)\right\}$$

$$= \frac{3\pi a^2}{4}$$

FP2 June 2008

1(a) $(x^2+y^2)^2 = 3xy^2$

(i) using polar coordinates

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$ $\qquad r^2 = x^2+y^2$

$\therefore (r^2)^2 = 3(r\cos\theta)(r\sin\theta)^2$

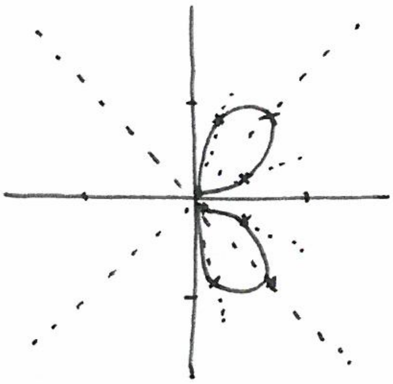
$\therefore r^4 = 3r^3\cos\theta\sin^2\theta$

$\therefore r = 3\cos\theta\sin^2\theta$

(ii)

θ	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
r	0	-1.06	0	1.06	0	1.06	0	-1.06	0

θ	$\frac{\pi}{8}$	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\frac{7\pi}{8}$
r	0.41	0.98	-0.98	-0.41



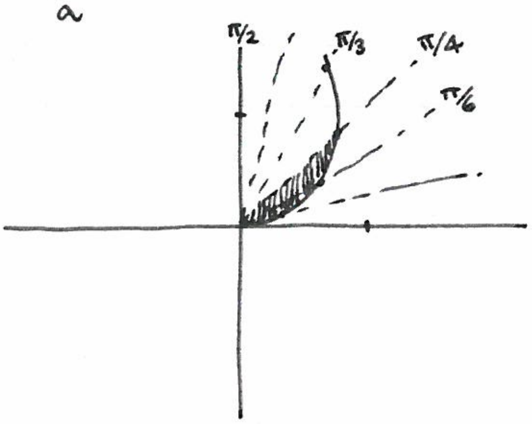
since $r > 0$ then only 2 parts

FP2 January 2009

3 (a) $r = a\tan\theta \qquad 0 \le \theta \le \pi/3 \qquad a > 0$

(i)

θ	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\frac{r}{a}$	0	0.27	0.58	1	1.73



(ii) area between curve and $\theta = \pi/4$ shaded on graph

$$= \int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/4} (a\tan\theta)^2 \, d\theta$$

$$= \frac{a^2}{2} \int_0^{\pi/4} \tan^2\theta \, d\theta$$

can integrate $\sec^2$ not $\tan^2$
$\tan^2\theta + 1 = \sec^2\theta$

$$= \frac{a^2}{2} \int_0^{\pi/4} (\sec^2\theta - 1) \, d\theta$$

$$= \frac{a^2}{2} \Big[\tan\theta - \theta \Big]_0^{\pi/4}$$

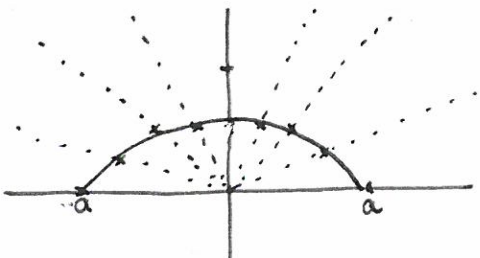
$$= \frac{a^2}{2} \left\{ \left(\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} \right) - (\tan 0 - 0) \right\}$$

$$= \frac{a^2}{2} \left(1 - \frac{\pi}{4} \right)$$

FP2 June 2009.

1 (b) $r = \dfrac{a}{1+\sin\theta}$ $\quad 0 \le \theta \le \pi \quad a > 0$

θ	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$	π
$\frac{r}{a}$	1	0.72	0.59	0.52	0.5	0.52	0.59	0.72	1



(should be symmetric)

(ii) For polar coordinates $x = r\cos\theta$, $\quad y = r\sin\theta$

$$r(1+\sin\theta) = a$$
$$\therefore r + r\sin\theta = a$$
$$\therefore r + y = a$$

Since $r^2 = (x)^2 + (y)^2 \qquad (r\cos\theta)^2 + (r\sin\theta)^2 = x^2+y^2$

$$r = \sqrt{x^2+y^2}$$
$$\therefore \sqrt{x^2+y^2} = a-y$$
$$\therefore x^2+y^2 = (a-y)^2$$
$$= a^2 + y^2 - 2ay$$
$$\therefore y^2 - y^2 + 2ay = a^2 - x^2$$
$$\therefore 2ay = a^2 - x^2$$
$$y = \frac{a^2-x^2}{2a}$$

FP2 January 2010

1(b) $x^2 + y^2 = xy + 1$

(i) If $x = r\cos\theta$, $y = r\sin\theta$ then

$$r^2\cos^2\theta + r^2\sin^2\theta = (r\cos\theta)(r\sin\theta) + 1$$

$$\therefore r^2 = r^2\cos\theta\sin\theta + 1$$

$$\therefore r^2 = r^2 \times \left(\tfrac{1}{2}\sin 2\theta\right) + 1$$

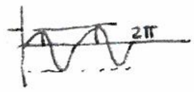
$$\therefore 2r^2 = r^2\sin 2\theta + 2$$

$$\therefore r^2(2 - \sin 2\theta) = 2$$

$$\therefore r^2 = \frac{2}{2 - \sin 2\theta}$$

(ii) $-1 \le \sin 2\theta \le 1$ if $\sin 2\theta = -1$ $r^2 = \dfrac{2}{2+1} = \dfrac{2}{3}$

$\therefore r = \sqrt{\frac{2}{3}}$ when $\theta = \frac{3\pi}{4}$ and when $\theta = \frac{7\pi}{4}$



If $\sin 2\theta = 1$ $r^2 = \dfrac{2}{2-1} = \dfrac{2}{1}$

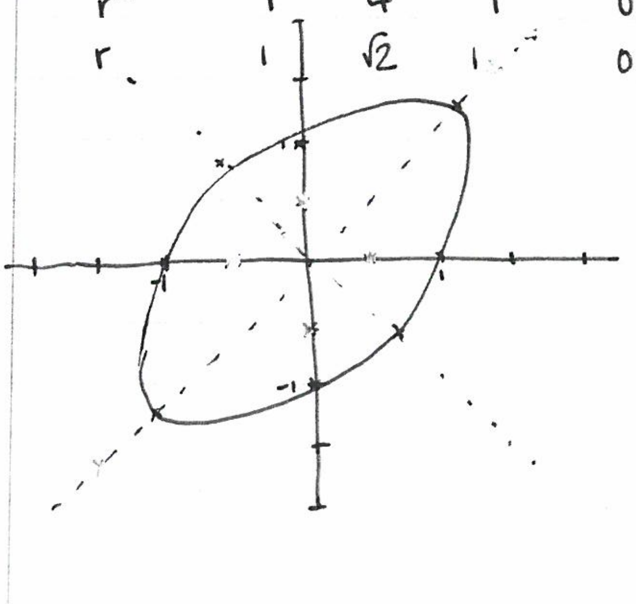
$\therefore r = 1$ when $\theta = \frac{\pi}{4}$ and when $\theta = \frac{5\pi}{4}$

So max value $r = 1$ $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

min value $r = \sqrt{\frac{2}{3}}$ $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

(iii)

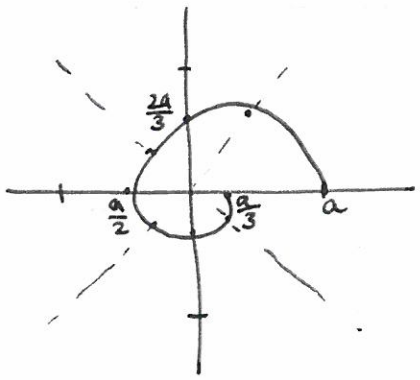
θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r^2	1	4	1	0.44	1	4	1	0.44	1
r	1	$\sqrt{2}$	1	0.82	1	1.41	1	0.82	1



FP2 June 2010.

1(b) $r = \dfrac{\pi a}{\pi + \theta}$ $a > 0$ $0 \leq \theta < 2\pi$

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\frac{r}{a}$	1	0.8	0.67	0.57	0.5	0.44	0.4	0.36	0.33



area of curve $0 \leq \theta \leq \pi$ and lines $\theta = 0$ and $\theta = \pi$

$$= \int_0^{\pi} \frac{1}{2} r^2 \, d\theta$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$= \int_0^{\pi} \frac{1}{2}\left(\frac{\pi a}{\pi + \theta}\right)^2 d\theta$$

$$= \frac{\pi^2 a^2}{2} \int_0^{\pi} \frac{1}{(\pi + \theta)^2} \, d\theta$$

$$= -\frac{\pi^2 a^2}{2} \left[\frac{1}{(\pi + \theta)}\right]_0^{\pi}$$

$$= -\frac{\pi^2 a^2}{2} \left[\frac{1}{2\pi} - \frac{1}{\pi}\right]$$

$$= \left(-\frac{\pi^2 a^2}{2}\right)\left(-\frac{1}{2\pi}\right)$$

$$= \frac{\pi a^2}{4}$$

(c) $$\int_0^{3/2} \frac{1}{9 + 4x^2} \, dx = \int_0^{3/2} \frac{1}{4\left(\frac{9}{4} + x^2\right)} \, dx$$

$$y = \arctan(x)$$
$$\frac{dy}{dx} = \frac{1}{1 + x^2}$$
$$y = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$$
$$\frac{dy}{dx} = \frac{1}{a^2 + x^2}$$

$$= \frac{1}{4} \int_0^{3/2} \frac{1}{\left(\frac{9}{4} + x^2\right)} \, dx$$

$$= \frac{1}{4} \int_0^{3/2} \frac{dx}{\left(\frac{3}{2}\right)^2 + x^2}$$

$$\therefore \int_0^{3/2} \frac{1}{9+4x^2}\,dx$$

$$= \frac{1}{4} \cdot \frac{2}{3} \cdot \left[\arctan\left(\frac{x}{(3/2)}\right) \right]_0^{3/2}$$

$$= \frac{1}{6} \arctan(1) - 0$$

$$= \frac{1}{6} \cdot \frac{\pi}{4}$$

$$= \frac{\pi}{24}$$

FP2 January 2011.

1 (a) $r = 2(\cos\theta + \sin\theta) \qquad -\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$

(i) cartesian equation

$$\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right\} \qquad r^2 = x^2 + y^2$$

$$r = 2(\cos\theta + \sin\theta)$$

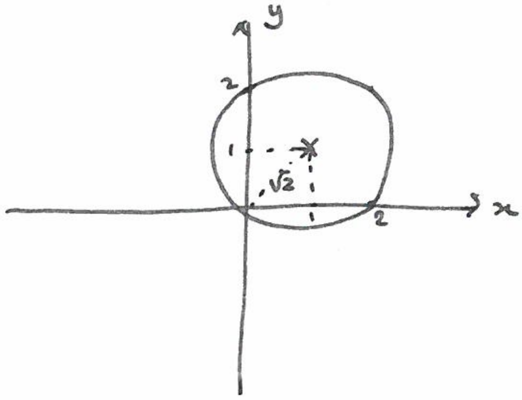
$$r^2 = r \times 2(\cos\theta + \sin\theta)$$

$$\therefore r^2 = 2r\cos\theta + 2r\sin\theta$$

$$\therefore r^2 = 2(r\cos\theta) + 2(r\sin\theta)$$

$$\therefore r^2 = 2x + 2y$$

$$\therefore x^2 + y^2 = 2x + 2y$$



$$\therefore x^2 - 2x + y^2 - 2y = 0$$

$$(x-1)^2 - 1 + (y-1)^2 - 1 = 0$$

$$\therefore (x-1)^2 + (y-1)^2 = 2$$

circle centre $(1, 1)$ radius $\sqrt{2}$.

(ii) region bounded $A = \int_0^{\pi/2} \frac{1}{2} r^2 \, d\theta$

$$= \int_0^{\pi/2} \frac{1}{2}\left[2(\cos\theta + \sin\theta)\right]^2 d\theta$$

$$= \int_0^{\pi/2} 2(\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta)\, d\theta$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$= \int_0^{\pi/2} 2(1 + 2\cos\theta\sin\theta)\, d\theta$$

$$= \int_0^{\pi/2} 2(1 + \sin 2\theta)\, d\theta$$

$$= 2\left[\left(\theta - \frac{1}{2}\cos 2\theta\right)\right]_0^{\pi/2}$$

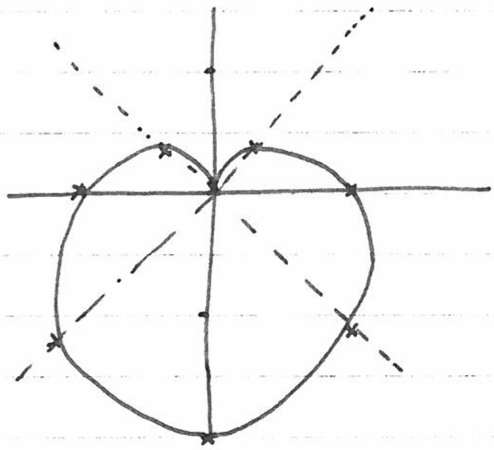
$$= 2\left(\left[\frac{\pi}{2} + \frac{1}{2}\right] - \left[0 - \frac{1}{2}\right]\right)$$

$$= \pi + 2.$$

FP2 June 2011

1(a)(i) $r = a(1 - \sin\theta)$ $a > 0$ $0 \le \theta < 2\pi$

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\frac{r}{a}$	1	0.29	0	0.29	1	1.71	2	1.71	1



(ii) $\text{Area} = \int_0^{2\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{2\pi} a^2 \frac{1}{2} (1 - \sin\theta)^2 \, d\theta$

$$= \frac{a^2}{2} \int_0^{2\pi} \left(1 + \sin^2\theta - 2\sin\theta\right) d\theta$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$$= \frac{a^2}{2} \int_0^{2\pi} \left(1 + \frac{1}{2} - \frac{1}{2}\cos 2\theta - 2\sin\theta\right) d\theta$$

$$= \frac{a^2}{2} \left[\frac{3}{2}\theta - \frac{1}{4}\sin 2\theta + 2\cos\theta\right]_0^{2\pi}$$

$$= \frac{a^2}{2} \left[(3\pi + 2) - (2)\right]$$

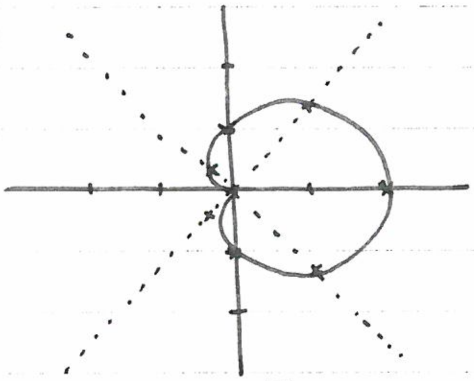
$$= \frac{3\pi a^2}{2}$$

FP2 January 2012

1(a) $r = 1 + \cos\theta \qquad 0 \le \theta < 2\pi$

(i)

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	2	1.71	1	0.29	0	0.29	1	1.71	2



(ii) area $= \int_0^{2\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{2\pi} \frac{1}{2}(1+\cos\theta)^2 \, d\theta$

$= \frac{1}{2}\int_0^{2\pi} \left(1 + \cos^2\theta + 2\cos\theta\right) d\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$

$= \frac{1}{2}\cos 2\theta + \frac{1}{2}$

$= \frac{1}{2}\int_0^{2\pi} \left(\frac{3}{2} + \frac{1}{2}\cos 2\theta + 2\cos\theta\right) d\theta$

$= \frac{1}{2}\left[\frac{3\theta}{2} + \frac{1}{4}\sin 2\theta + 2\sin\theta\right]_0^{2\pi}$

$= \frac{1}{2}\left[(3\pi + 0 + 0) - (0 + 0 + 0)\right]$

$= \frac{3\pi}{2}$