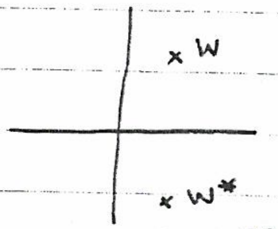
FP2 January 2006.

2. $0 < \theta < \pi/6$ $\quad w = \frac{1}{2}e^{3j\theta}$

(i) $w = \frac{1}{2}e^{3j\theta} = \frac{1}{2}(\cos 3\theta + j\sin 3\theta)$



$0 < 3\theta < \pi/2$

$\therefore$ modulus $= |w| = \frac{1}{2}$

arg $(w) = 3\theta$

$w^* = \frac{1}{2}e^{-3j\theta} = \frac{1}{2}\left(\cos(-3\theta) + j\sin(-3\theta)\right)$

$\therefore |w^*| = \frac{1}{2} \quad \arg(w^*) = -3\theta$

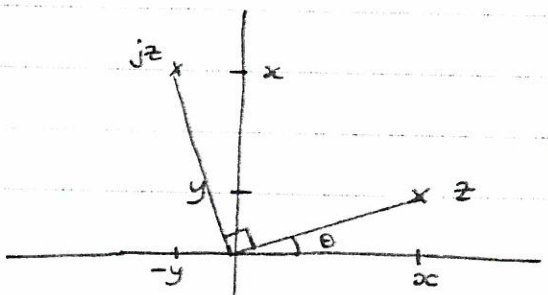
$jw = \frac{1}{2}je^{3j\theta} \qquad |jw|$

$= \frac{1}{2}j(\cos 3\theta + j\sin 3\theta)$

$= \frac{1}{2}(j\cos 3\theta + j^2\sin 3\theta)$

$= \frac{1}{2}(-\sin 3\theta + j\cos 3\theta)$

$= \frac{1}{2}\left(\cos(3\theta + \pi/2) + j\sin(3\theta + \pi/2)\right)$



$z = x + jy$

$jz = jx + j^2y = -y + jx$

(ii) $(1+w)(1+w^*) = \frac{5}{4} + \cos 3\theta$

If $w = \frac{1}{2}e^{3j\theta}$

$\frac{e^{3j\theta}}{e^{3j\theta}} = 1$

$(1+w)(1+w^*) = (1+\frac{1}{2}e^{3j\theta})(1+\frac{1}{2}e^{-3j\theta})$

$= 1 + \frac{1}{2}e^{3j\theta} + \frac{1}{2}e^{-3j\theta} + \frac{1}{4}e^{3j\theta}e^{-3j\theta}$

$= 1 + \frac{1}{2}(\cos 3\theta + j\sin 3\theta) + \frac{1}{2}(\cos 3\theta - j\sin 3\theta) + \frac{1}{4}$

$= \frac{5}{4} + \frac{1}{2}\cos 3\theta + \frac{1}{2}j\sin 3\theta + \frac{1}{2}\cos 3\theta - \frac{1}{2}j\sin 3\theta$

$= \frac{5}{4} + \cos 3\theta.$

(iii) $C = \cos 2\theta - \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 8\theta - \frac{1}{8}\cos 11\theta + \ldots$

$S = \sin 2\theta - \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 8\theta - \frac{1}{8}\sin 11\theta + \ldots$

$C + j\sin\theta = (\cos 2\theta + j\sin 2\theta) - \frac{1}{2}(\cos 5\theta + j\sin 5\theta) + \frac{1}{4}(\cos 8\theta + j\sin 8\theta)\ldots$

$= e^{2j\theta} - \frac{1}{2}e^{5j\theta} + \frac{1}{4}e^{8j\theta} - \frac{1}{8}e^{11j\theta}\ldots$

geometric (progression) series where $a = e^{2j\theta}$ $\quad r = -\frac{1}{2}e^{3j\theta}$

$$\therefore\ C + jS = \frac{a}{1-r} = \frac{e^{2j\theta}}{1+\frac{1}{2}e^{3j\theta}} = \frac{e^{2j\theta}}{\left(1+\frac{1}{2}e^{3j\theta}\right)} \times \frac{\left(1+\frac{1}{2}e^{-3j\theta}\right)}{\left(1+\frac{1}{2}e^{-3j\theta}\right)}$$

$$\therefore\ C + jS = \frac{e^{2j\theta} + \frac{1}{2}e^{-j\theta}}{\frac{5}{4} + \cos 3\theta} = \frac{4\left(e^{2j\theta} + \frac{1}{2}e^{-j\theta}\right)}{(5 + 4\cos 3\theta)}$$

$$\therefore\ C + jS = \frac{4e^{2j\theta} + 2e^{-j\theta}}{5 + 4\cos 3\theta} = \frac{4\cos 2\theta + 4j\sin 2\theta + 2\cos\theta - 2j\sin\theta}{5 + 4\cos 3\theta}$$

Real : $\quad C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$

Imag : $\quad S = \dfrac{4\sin 2\theta - 2\sin\theta}{5 + 4\cos 3\theta}$

FP2 June 2006

2(a)(i) $z = \cos\theta + j\sin\theta$, $z^n = (\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$

$$\frac{1}{z^n} = z^{-n} = (\cos\theta + j\sin\theta)^{-n} = \cos(-n\theta) + j\sin(-n\theta)$$
$$= \cos n\theta - j\sin n\theta$$

$$\therefore\ z^n + \frac{1}{z^n} = \cos n\theta + j\sin n\theta + \cos n\theta - j\sin n\theta$$

$$\therefore\ z^n + \frac{1}{z^n} = 2\cos n\theta$$

$$z^n - \frac{1}{z^n} = \cos n\theta + j\sin n\theta - \cos n\theta + j\sin n\theta$$

$$\therefore\ z^n - \frac{1}{z^n} = 2j\sin n\theta .$$

(ii) $$\left[z - \frac{1}{z}\right]^4\left[z + \frac{1}{z}\right]^2 = (2j\sin\theta)^4\ [2\cos\theta]^2$$
$$= 16\sin^4\theta \times 2^2\cos^2\theta$$
$$= 64\sin^4\theta\cos^2\theta$$

But $\left[z-\frac{1}{z}\right]^4 = \left[z-\frac{1}{z}\right]^2\left[z-\frac{1}{z}\right]^2$ and $\left[z-\frac{1}{z}\right]^2\left[z+\frac{1}{z^2}\right] = \left[z^2-\frac{1}{z^2}\right]$

$$\therefore\ \left[z-\frac{1}{z}\right]^2\left[z^2-\frac{1}{z^2}\right] = z^6 - 2z^2 + \frac{1}{z^2} - 2z^4 + 4 - \frac{2}{z^4} + z^2 - \frac{2}{z^2} + \frac{1}{z^6}$$

$$\therefore\ \left[z-\frac{1}{z}\right]^4\left[z+\frac{1}{z}\right]^2 = z^6 - 2z^4 - z^2 + 4 - \frac{1}{z^2} - \frac{2}{z^4} + \frac{1}{z^6}$$

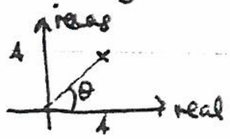
$$= \left(z^6 + \frac{1}{z^6}\right) - 2\left(z^4 + \frac{1}{z^4}\right) - \left(z^2 + \frac{1}{z^2}\right) + 4$$
$$= 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$$

$$\therefore\ \sin^4\theta\cos^2\theta = \frac{1}{64}\left\{2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4\right\}$$

$$\therefore\ \sin^4\theta\cos^2\theta = \frac{1}{32}\cos 6\theta - \frac{1}{16}\cos 4\theta - \frac{1}{32}\cos 2\theta + \frac{1}{16}$$

$$\therefore\ A = \frac{1}{32},\quad B = -\frac{1}{16},\quad C = -\frac{1}{32},\quad D = \frac{1}{16}.$$

(b) (i) $z = 4 + 4j$ mod $z = |z| = |4+4j| = \sqrt{4^2+4^2} = \sqrt{32}$



$\tan\theta = \frac{4}{4}$ $\therefore\ \theta = \pi/4$

$\therefore\ \arg(4+4j) = \pi/4$

(ii) 5^{th} roots of $4+4j$ in form $re^{j\theta}$ $r>0$ and $-\pi < \theta \leq \pi$

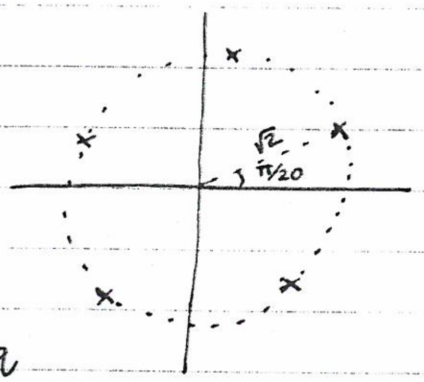
$$4+4j = \sqrt{32}\, e^{j\theta} = \sqrt{32}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) \Rightarrow$$

All roots have modulus $\sqrt[5]{32} = \sqrt{2}$

For the arguments, θ, let $\theta = \dfrac{\phi + 2k\pi}{n}$ where $\phi = \dfrac{\pi}{4}$, $n = 5$

The principal argument $\theta = \dfrac{\frac{\pi}{4} + 2k\pi}{5} = \dfrac{\pi}{20} + \dfrac{2k\pi}{5}$ $k = 0,1,2,3,4$

$\therefore \theta = \dfrac{\pi}{20}, \dfrac{9\pi}{20}, \dfrac{17\pi}{20}, -\dfrac{7\pi}{20}\ -\dfrac{3\pi}{4}$



(iii) $(p+qj)^5 = 4+4j$

5th root $(4+4j)^{1/5} = \sqrt{2}\, e^{j\theta}$

If we want integer values for p and q then they must be a multiple of $\sqrt{2}$; only $\theta = -\dfrac{3\pi}{4}$ is able to give this $e^{-\frac{3\pi}{4}j} = \cos\left(-\dfrac{3\pi}{4}\right) + j\sin\left(-\dfrac{3\pi}{4}\right)$

$$= -\frac{1}{\sqrt{2}} + j\left(-\frac{1}{\sqrt{2}}\right)$$

$\therefore (p+qj)^5 = 4+4j \Rightarrow \sqrt{2}\left(-\dfrac{1}{\sqrt{2}} - j\dfrac{1}{\sqrt{2}}\right) = -1-j$

$\therefore p = -1,\ q = -1.$

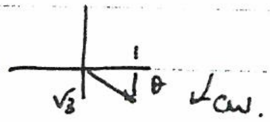
FP2 January 2007.

2(a) $w = 3e^{-\pi j/12}$ and $z = 1 - \sqrt{3}j$

(i) $w = 3\left(\cos\left(-\frac{\pi}{12}\right) + j\sin\left(\frac{-\pi}{12}\right)\right) \Rightarrow |w| = 3 \quad \arg(w) = \left(\frac{-\pi}{12}\right)$

$z = 1 - \sqrt{3}j \Rightarrow |z| = \sqrt{1+3} = \sqrt{4} = 2$

$\tan\theta = \sqrt{3} \quad \therefore \theta = -\frac{\pi}{3}$ (cw angle)



$\frac{w}{z}$ for modulus → divide by mod $\left|\frac{w}{z}\right| = \frac{3}{2}$

for arguments → subtract $\arg\left(\frac{w}{z}\right) = \arg(w) - \arg(z)$

$= -\frac{\pi}{12} - \left(-\frac{\pi}{3}\right)$

$= \frac{3\pi}{12} = \frac{\pi}{4}$

(ii) $\frac{w}{z} = \frac{3}{2}\left(\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right)$

$\therefore \frac{w}{z} = \frac{3}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}j \qquad a = \frac{3}{2\sqrt{2}}, \quad b = \frac{3}{2\sqrt{2}}$

(b) $n > 0 \qquad 0 < \theta < \frac{\pi}{n}$

(i) $e^{-\frac{1}{2}j\theta} + e^{\frac{1}{2}j\theta} = \left[\cos\left(\tfrac{1}{2}\theta\right) - j\sin\left(\tfrac{1}{2}\theta\right)\right] + \left[\cos\left(\tfrac{1}{2}\theta\right) + j\sin\left(\tfrac{1}{2}\theta\right)\right]$

$= 2\cos\left(\tfrac{1}{2}\theta\right)$

$1 + e^{j\theta} = \frac{e^{-\frac{1}{2}j\theta} + e^{\frac{1}{2}j\theta}}{e^{-\frac{1}{2}j\theta}} = \frac{e^{-\frac{1}{2}j\theta}}{e^{-\frac{1}{2}j\theta}} + \frac{e^{\frac{1}{2}j\theta}}{e^{-\frac{1}{2}j\theta}} = 1 + e^{j\theta}$

$\therefore 1 + e^{j\theta} = \frac{2\cos\left(\frac{1}{2}\theta\right)}{e^{-\frac{1}{2}j\theta}} = 2e^{\frac{1}{2}j\theta}\cos\left(\tfrac{1}{2}\theta\right)$

(iii) $C = 1 + \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta + \binom{n}{3}\cos 3\theta + \ldots + \binom{n}{n}\cos n\theta$

$\therefore S = \binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta + \binom{n}{3}\sin 3\theta + \ldots + \binom{n}{n}\sin n\theta$

$C + iS = 1 + \binom{n}{1}\left[\cos\theta + j\sin\theta\right] + \binom{n}{2}\left[\cos 2\theta + j\sin 2\theta\right] + \ldots \binom{n}{n}\left[\cos n\theta + j\sin n\theta\right]$

$= 1 + \binom{n}{1}e^{j\theta} + \binom{n}{2}e^{2j\theta} + \ldots + \binom{n}{n}e^{nj\theta} \qquad a = 1, \; r = e^{j\theta}$

$\therefore C + iS = \left(1 + e^{j\theta}\right)^n$

$= \left(2e^{\frac{1}{2}j\theta}\cos\left(\tfrac{1}{2}\theta\right)\right)^n$

$= 2^n e^{\frac{n}{2}j\theta}\cos^n\left(\tfrac{1}{2}\theta\right) = 2^n\left(\cos\left(\frac{n\theta}{2}\right) + j\sin\left(\frac{n\theta}{2}\right)\right)\cos^n\left(\frac{\theta}{2}\right)$

cf real: $C = 2^n \cos\left(\frac{n\theta}{2}\right) \cos^n\left(\frac{\theta}{2}\right)$

cf imag: $S = 2^n \sin\left(\frac{n\theta}{2}\right) \cos^n\left(\frac{\theta}{2}\right)$

$$\therefore \frac{S}{C} = \frac{2^n \sin\left(\frac{n\theta}{2}\right) \cos^n\left(\frac{\theta}{2}\right)}{2^n \cos\left(\frac{n\theta}{2}\right) \cos^n\left(\theta/2\right)} = \frac{\sin\left(\frac{n\theta}{2}\right)}{\cos\left(\frac{n\theta}{2}\right)} = \tan\left(\frac{n\theta}{2}\right)$$

FP2 June 2007.

2 (a) Show $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$

$(\cos\theta + j\sin\theta)^5 = (c+js)^5$ 1 5 10 10 5 1

$= c^5 + 5c^4 js + 10c^3(js)^2 + 10c^2(js)^3 + 5c(js)^4 + (js)^5$

$= c^5 + 5c^4 js - 10c^3s^2 - 10c^2 js^3 + 5cs^4 + js^5$

but $(\cos\theta + j\sin\theta)^5 = \cos 5\theta + j\sin 5\theta$ (De Moivre)

cf real parts $\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$

cf imag. parts $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$

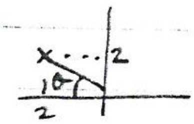
$\therefore \sin 5\theta = 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta$

$= 5\sin\theta(1 - 2\sin^2\theta + \sin^4\theta) - 10\sin^3\theta + 10\sin^5\theta + \sin^5\theta$

$= 5\sin\theta - 10\sin^3\theta + 5\sin^5\theta - 10\sin^3\theta + 10\sin^5\theta + \sin^5\theta$

$= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$

(b) (i) Cube roots of $-2+2j$ $r>0$ and $-\pi < \theta \le \pi$



$|-2+2j| = \sqrt{4+4} = \sqrt{8}$

$\tan\theta = \frac{2}{2} = 1$ $\arg(-2+2j) = \frac{3\pi}{4}$

$\therefore (-2+2j) = \sqrt{8}\, e^{\frac{3\pi}{4}j}$

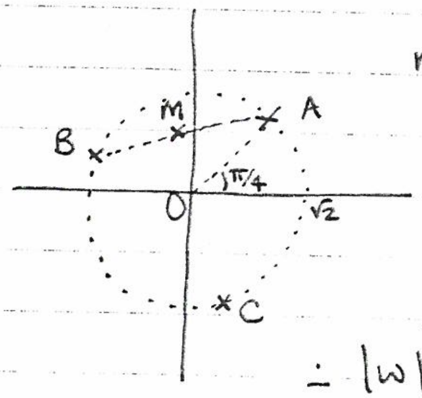
For cube root $r = \sqrt[3]{8} = \sqrt{2}$ $\theta = \dfrac{\frac{3\pi}{4} + 2k\pi}{3} = \frac{3\pi}{12} = \frac{\pi}{4}$ $(k=0)$

Also $\theta = \dfrac{\frac{3\pi}{4} + 2\pi}{3} = \frac{11\pi}{12}$

and $\theta = \frac{3\pi}{4} + \frac{4\pi}{3} = \frac{19\pi}{12} = -\frac{5\pi}{12}$

$\therefore (-2+2j)^{1/3} = \sqrt{2}\, e^{\frac{\pi}{4}j}$

(ii)



M represents w: midpoint of AB.

$A = \sqrt{2}\, e^{3\pi/12}$ $B = \sqrt{2}\, e^{\frac{11\pi}{12}}$

By vectors, $\underline{w} = \frac{1}{2}(\underline{A} + \underline{B})$

or $\underline{w} = \frac{1}{2}(\sqrt{2}\, e^{\pi/4} + \sqrt{2}\, e^{11\pi/12})$

$\therefore \underline{w} = \frac{1}{2}\sqrt{2}\left\{\cos(\pi/4) + \cos(\frac{11\pi}{12}) + i\left(\sin\frac{\pi}{4} + \sin\frac{11\pi}{12}\right)\right\}$

$\therefore |w| = \frac{\sqrt{2}}{2}$

The argument is angle to $\underline{w} = \frac{1}{2}\left(\frac{\pi}{4} + \frac{11\pi}{12}\right)$ (halfway between AOB.)

$\therefore$ argument $= \frac{1}{2}\left(\frac{7}{6}\pi\right) = \frac{7\pi}{12}$ and $|w| = \frac{1}{\sqrt{2}}$

(iv) Find w^6 in form $a + bj$

$|w^6| = \left(\frac{1}{\sqrt{2}}\right)^6 = \frac{1}{8}$

If you multiply a complex number by another, you multiply their moduli and add their arguments

$\arg(w^6) = 6 \times \frac{7\pi}{12} = \frac{7\pi}{2}$

$\therefore w^6 = \frac{1}{8}\left(\cos\left(\frac{7\pi}{2}\right) + j\sin\left(\frac{7\pi}{2}\right)\right)$

$\therefore w^6 = \frac{1}{8}\left(0 + j(-1)\right)$

$\therefore w^6 = -\frac{1}{8}j.$

FP2 January 2008

2 (a) The 4th roots of $16j$ in form $re^{j\theta}$ $r>0$ and $-\pi < \theta \leq \pi$

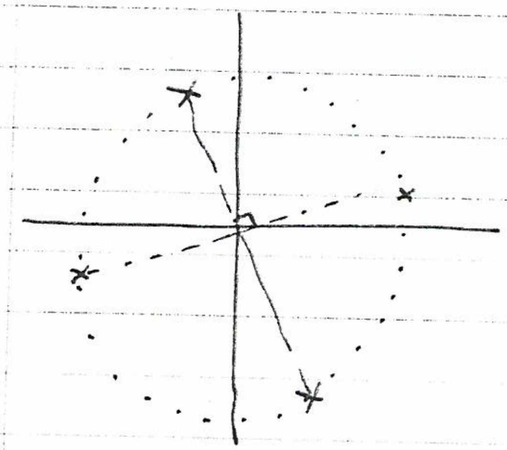
$r = (16)^{1/4} = 2$ $\quad$ $16j = 16(\cos\theta + j\sin\theta)$

$\phi = \frac{\pi}{2}$ $\quad$ $\cos\theta = 0 \Rightarrow \theta = \pi/2$

For 4th roots $\quad$ $\sin \pi/2 = 1 \Rightarrow j$

$$\theta = \frac{\frac{\pi}{2} + 2\pi k}{4} \qquad k = 0, 1, 2, ..$$

$$\therefore \theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}\left(= -\frac{\pi}{8}\right), \frac{13\pi}{8}\left(= -\frac{3\pi}{8}\right)$$



(b) (i) $(1-2e^{j\theta})(1-2e^{-j\theta}) = 1 - 2e^{-j\theta} - 2e^{j\theta} + 4e^{j\theta}.e^{-j\theta}$

$= 1 - 2e^{-j\theta} - 2e^{j\theta} + 4.$

$= 5 - 2(e^{j\theta} + e^{-j\theta})$ $\quad$ but $\frac{e^{j\theta}+e^{-j\theta}}{2} = \cos\theta$

$= 5 - 2 \times 2\cos\theta$

$= 5 - 4\cos\theta$ $\quad$ $\therefore e^{j\theta} + e^{-j\theta} = 2\cos\theta$

(ii) $C = 2\cos\theta + 4\cos 2\theta + 8\cos 3\theta + ... + 2^n \cos n\theta$

$S = 2\sin\theta + 4\sin 2\theta + 8\sin 3\theta + ... + 2^n \sin n\theta$

$\therefore C + jS = 2(\cos\theta + j\sin\theta) + 4(\cos 2\theta + j\sin 2\theta) + ... + 2^n(\cos n\theta + j\sin n\theta)$

$\therefore C + jS = 2e^{j\theta} + 4e^{2j\theta} + 8e^{3j\theta} + ... + 2^n e^{nj\theta}$

this is a gp where $a = 2e^{j\theta}$ $\quad r = 2e^{j\theta}$ $\quad$ using $S_\infty = \frac{a}{(1-r)}$

$S_n = \frac{a(1-r^n)}{(1-r)}$

$$C + jS = \frac{2e^{j\theta}(1-(2e^{j\theta})^n)}{(1-2e^{j\theta})}$$

$$= \frac{2e^{j\theta}(1-2^n e^{nj\theta})(1-2e^{-j\theta})}{(1-2e^{j\theta})(1-2e^{-j\theta})}$$

$$\therefore\ c + js = \frac{\left(2e^{j\theta} - 2^{n+1} e^{(n+1)j\theta}\right)\left(1 - 2e^{-j\theta}\right)}{5 - 4\cos\theta}$$

$$= \frac{2e^{j\theta} - 4 - 2^{n+1} e^{(n+1)j\theta} + 2^{n+2} e^{nj\theta}}{5 - 4\cos\theta}$$

equating real coeffs

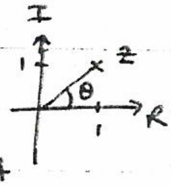
$$C = \frac{2\cos\theta - 4 - 2^{n+1}\cos(n+1)\theta + 2^{n+2}\cos n\theta}{5 - 4\cos\theta}$$

equating imag coeffs.

$$S = \frac{2\sin\theta - 4 - 2^{n+1}\sin(n+1)\theta + 2^{n+2}\sin n\theta}{5 - 4\cos\theta}$$

FP2 June 2008.

2. $z = \sqrt{32}\,(1+j)$ $\quad w = 8\left(\cos\left(\frac{7\pi}{12}\right) + j\sin\left(\frac{7\pi}{12}\right)\right)$



(i) $|z| = \sqrt{32+32} = 8$

$\tan\theta = 1 \qquad \arg(z) = \pi/4$ $\qquad \tan^{-1}(1) = \pi/4$

$|z^*| = \sqrt{32+32} = 8$ $\qquad z^* = \sqrt{32}\,(1-j)$

$\tan\theta = -1 \qquad \arg(z^*) = -\pi/4$

$|zw| = 8 \times 8 = 64$ $\qquad$ When complex numbers ×

$\arg(zw) = \frac{\pi}{4} + \frac{7\pi}{12} = \frac{10\pi}{12}$ $\qquad \Rightarrow$ moduli × and

$\Rightarrow$ arg +

$\therefore \arg(zw) = \frac{5\pi}{6}$ $\qquad |w| = 8 \quad \arg(w) = \left(\frac{7\pi}{12}\right)$

$\left|\frac{z}{w}\right| = \frac{8}{8} = 1$ $\qquad$ when divided $\Rightarrow$ mod ÷

$\Rightarrow$ arg subtract.

$\arg\left(\frac{z}{w}\right) = \frac{\pi}{4} - \frac{7\pi}{12} = -\frac{4\pi}{12}$

$\therefore \arg\left(\frac{z}{w}\right) = -\frac{\pi}{3}$

(ii) $\left(\frac{z}{w}\right) = a + bj = 1\left(\cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)\right)$

$= 1\left(\frac{1}{2} + j\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= \frac{1}{2} - \frac{\sqrt{3}}{2}j$

(iii) cube roots of z

modulus $= \sqrt[3]{8} = 2$

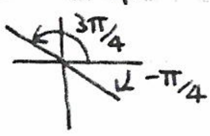
for argument $\theta = \frac{\phi + 2\pi k}{n} = \frac{\frac{\pi}{4} + 2\pi k}{3}$ $\qquad k = 0, 1, 2,$

$\therefore \theta = \frac{\pi}{12}, \quad \frac{\pi}{12} + \frac{2\pi}{3} = \frac{9\pi}{12} = \frac{3\pi}{4}, \quad \frac{\pi}{12} + \frac{4\pi}{3} = \frac{17\pi}{12} = -\frac{7\pi}{12}$

$\omega_0 = 2e^{\pi j/12}, \quad 2e^{3\pi j/4}, \quad 2e^{-\frac{7\pi j}{12}}$

(iv) $w = 8e^{\frac{7\pi}{12}j}, \quad w^* = 8e^{-\frac{7\pi}{12}j}$ $\qquad \therefore 2e^{-\frac{7\pi j}{12}} = \frac{1}{4}w^*$ $\qquad$ so $k_1 = \frac{1}{4}$

$z^* = 8e^{j\pi/4}$



Since $e^{\frac{-j\pi}{4}} = a - ib$ $\qquad e^{\frac{3\pi j}{4}} = -a + ib$

$\therefore e^{\frac{-\pi i}{4}} = - e^{\frac{3\pi i}{4}}$

$\therefore z^* = 8e^{-j\pi/4} = -8e^{j3\pi/4}$ $\qquad \therefore 2e^{\frac{3\pi j}{4}} = -\frac{1}{4} z^*$ so $k_2 = -\frac{1}{4}$

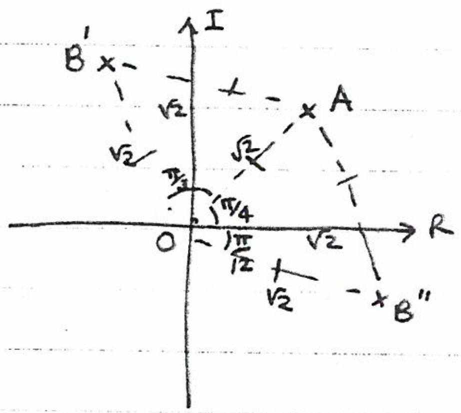
$jw = 8j e^{7\pi/12 j} = 8 e^{\frac{\pi j}{2}} . e^{\frac{7\pi}{12}j} = 8 e^{\frac{13\pi}{12}j} = -8 e^{+\pi/12 j}$

$\therefore 2e^{\frac{\pi j}{12}} = -\frac{1}{4} jw$ so $k_3 = -\frac{1}{4}$

FP2 January 2009.

2(i) $e^{j\pi/3}$ has modulus $= 1$

argument $= \frac{\pi}{3}$



(ii) $a = \sqrt{2}(1+j)$

$\therefore a \Rightarrow$ modulus $\sqrt{2}$

argument $= \pi/4$

$\therefore a = \sqrt{2}\, e^{\pi j/4}$

Equilateral triangle angles $\frac{\pi}{3}$ so $AOB' = \pi/3$, $\arg B' = \frac{\pi}{3} + \frac{\pi}{4}$

$AOB'' = \frac{\pi}{3}$, $\arg B'' = \frac{\pi}{3} - \frac{\pi}{4}$

$\therefore B' \to \text{mod}\,(\sqrt{2})^2 = 2 \quad \arg\left(\frac{7\pi}{12}\right) \qquad B' = 2e^{\frac{7\pi j}{12}}$

& $B'' \to \text{mod}\,(\sqrt{2})^2 = 2 \quad \arg\left(-\frac{\pi}{12}\right) \qquad B'' = 2e^{-\frac{\pi j}{12}}$

(iii) $z_1 = \sqrt{2}\, e^{\pi j/3} \qquad z_1^6 = (\sqrt{2})^6 (e^{\pi j/3})^6 = 8\, e^{2\pi j} \quad \left[e^{2\pi j} = \cos 2\pi + i\sin 2\pi\right]$

$= 1 + 0$

$\therefore z_1^6 = 8 \quad \Rightarrow re^{j\theta} = 8\, e^{0j} \quad (r = 8,\ \arg = 0)$

The six roots are based on modulus $= r = \sqrt{2}$

argument $\theta = \frac{\phi + 2\pi k}{n} = \frac{0 + 2\pi k}{6} \qquad k = 0, 1, 2, \ldots 5$

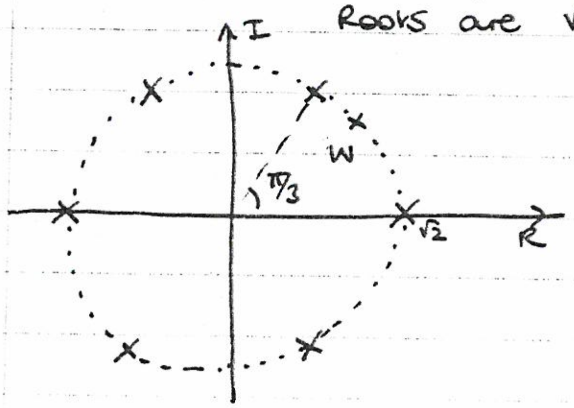
$\therefore \theta = \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{6\pi}{6}, \frac{8\pi}{6}, \frac{10\pi}{6}$

$\therefore \theta = 0, \frac{1}{3}\pi, \frac{2}{3}\pi, \pi, -\frac{2}{3}\pi, -\frac{1}{3}\pi$

Roots are $\sqrt{2}e^{0j}$, $\sqrt{2}e^{\frac{\pi}{3}j}$, $\sqrt{2}e^{j2\pi/3}$ etc.

Roots are $\pi/3$ apart on circle ϕ of $\sqrt{2}$

(iv)



let $w = z_1 e^{-j\pi/12}$

$\therefore w = \sqrt{2}\, e^{\pi j/3} \cdot e^{-j\pi/12}$

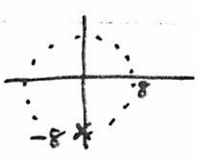
$\therefore w = \sqrt{2}\, e^{\pi j/4} \qquad r = \sqrt{2}$

$\therefore w = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right)$

$\therefore w = 1 + j \qquad \theta = \frac{\pi}{4}$

(v) $w^6 = \left(\sqrt{2}\, e^{\pi j/4}\right)^6 = (\sqrt{2})^6\, e^{6\pi j/4}$

$\therefore w^6 = 8\, e^{3\pi j/2} = -8j$



FP2 June 2009.

3. (b) $C = \cos\theta + \frac{1}{3}\cos 3\theta + \frac{1}{9}\cos 5\theta + \ldots$

$S = \sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{9}\sin 5\theta + \ldots$

$\therefore\ C + iS = (\cos\theta + i\sin\theta) + \frac{1}{3}(\cos 3\theta + i\sin 3\theta) + \frac{1}{9}(\cos 5\theta + i\sin 5\theta) \ldots$

$= e^{j\theta} + \frac{1}{3}e^{j3\theta} + \frac{1}{9}e^{j5\theta} + \ldots$ g.p $a = e^{j\theta}$, $r = \frac{1}{3}e^{2\theta j}$

$$S_\infty = \frac{a}{(1-r)} = \frac{e^{j\theta}}{(1-\frac{1}{3}e^{2\theta j})}$$

However $(1-\frac{1}{3}e^{2\theta j})(1-\frac{1}{3}e^{-2\theta j}) = 1 - \frac{1}{3}e^{-2\theta} - \frac{1}{3}e^{2\theta} + \frac{1}{9}$

$= \frac{10}{9} - \frac{1}{3}e^{-2\theta} - \frac{1}{3}e^{2\theta}$

$$\therefore\ S_\infty = C + iS = \frac{e^{j\theta}(1-\frac{1}{3}e^{-2\theta j})}{(1-\frac{1}{3}e^{2\theta j})(1-\frac{1}{3}e^{-2\theta j})}$$

$$= \frac{e^{j\theta} - \frac{1}{3}e^{-j\theta}}{\frac{1}{9}\{10 - 3(\cos 2\theta - i\sin 2\theta) - 3(\cos 2\theta + i\sin 2\theta)\}}$$

$$= \frac{\frac{1}{3}(3e^{j\theta} - e^{-j\theta})}{\frac{1}{9}\{10 - 6\cos 2\theta\}}$$

$e^{j\theta}$

$$= \frac{3\ (3\cos\theta + 3i\sin\theta - \cos\theta + i\sin\theta)}{2\ (5 - 3\cos 2\theta)}$$

$$= \frac{6\cos\theta + 12i\sin\theta}{2(5 - 3\cos 2\theta)}$$

$$= \frac{3\cos\theta + 6i\sin\theta}{5 - 3\cos 2\theta}$$

compare real parts

$$\therefore\ C = \frac{3\cos\theta}{5 - 3\cos 2\theta}$$

Compare imag. parts

$$\therefore\ S = \frac{6\sin\theta}{5 - 3\cos 2\theta}$$

FP2 January 2010

2(a) $\cos 5\theta \equiv a\cos^5\theta + b\cos^3\theta + c\cos\theta$ $\qquad c = \cos\theta$

$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ $\qquad s = \sin\theta$

$= c^5 + 5ic^4s + 10i^2c^3s^2 + 10i^3c^2s^3 + 5i^4cs^4 + i^5s^5$

comparing real parts

$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$

$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$

$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta + 5\cos^5\theta - 10\cos^3$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

$\therefore a = 16, \quad b = -20, \quad c = 5$

(b) $C = \cos\theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \dots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right)$

$S = \sin\theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \dots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right)$

$\therefore C + iS = (\cos\theta + i\sin\theta) + \left(\cos\left(\theta + \frac{2\pi}{n}\right) + i\sin\left(\theta + \frac{2\pi}{n}\right)\right) + \dots$

$+ \left(\cos\left(\theta + \frac{(2n-2)\pi}{n}\right) + i\sin\left(\theta + \frac{(2n-2)\pi}{n}\right)\right)$

$\therefore C + iS = e^{j\theta} + e^{(\theta + \frac{2\pi}{n})j} + \dots + e^{(\theta + (2n-2)\frac{\pi}{n})j}$

g.p. with $a = e^{j\theta}$ $\quad r = e^{\frac{2\pi}{n}j}$

$S_n = \dfrac{a(1-r^n)}{(1-r)} = \dfrac{e^{j\theta}\left(1 - \left(e^{\frac{2\pi}{n}j}\right)^n\right)}{\left(1 - e^{\frac{2\pi}{n}j}\right)} = \dfrac{e^{j\theta}\left(1 - e^{2\pi j}\right)}{\left(1 - e^{\frac{2\pi j}{n}}\right)}$

However, $e^{2\pi j} = 1(\cos(2\pi) + j\sin(2\pi)) = 1$

$\therefore S_n = \dfrac{e^{j\theta}(1-1)}{\left(1 - e^{\frac{2\pi}{n}j}\right)} = 0$

$\therefore C + iS = S_n = 0$

$\therefore$ equating real parts $C = 0$

equating imag parts $S = 0$

FP2 June 2010.

2(a) $z = \cos\theta + j\sin\theta$ $\qquad \frac{1}{z} = z^{-n}$

$z^n = (\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$

$z^{-n} = (\cos\theta + j\sin\theta)^{-n} = \cos n\theta - j\sin n\theta$

$\therefore \quad z^n + \frac{1}{z^n} = 2\cos n\theta$

$z^n - \frac{1}{z^n} = 2j\sin n\theta$

$\sin^5\theta \equiv A\sin\theta + B\sin 3\theta + C\sin 5\theta$ $\qquad$? choose "−" for sin

$$\left(z - \frac{1}{z}\right)^5 \equiv z^5 - 5\frac{z^4}{z} + 10\frac{z^3}{z^2} - 10\frac{z^2}{z^3} + 5\frac{z}{z^4} - \frac{1}{z^5}$$

$$= z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$$

$$= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$$

$$= 2j\sin 5\theta - 10j\sin 3\theta + 20j\sin\theta$$

but $\left(z - \frac{1}{z}\right)^5 = (2j\sin\theta)^5 = 32j^5\sin^5\theta = 32j\sin^5\theta$

$\therefore \quad \sin^5\theta = \frac{1}{32j}\left(20j\sin\theta - 10j\sin 3\theta + 2j\sin 5\theta\right)$

$\therefore \quad \sin^5\theta = \frac{5}{8}\sin\theta - \frac{5}{16}\sin 3\theta + \frac{1}{16}\sin 5\theta$

$\therefore \quad A = \frac{5}{8}, \quad B = -\frac{5}{16}, \quad C = \frac{1}{16}$

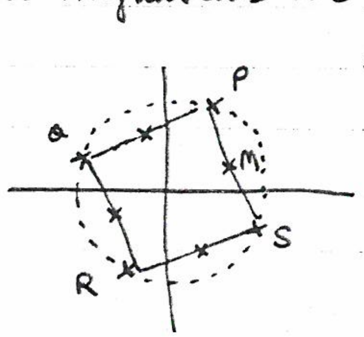
(b)(i) 4^{th} roots of $-9j$ $\qquad -9j \equiv 9e^{-\frac{\pi}{2}j} = 9e^{\frac{3\pi}{2}j}$

$\therefore \quad r = 9^{1/4} = \sqrt{3}$ $\qquad$ since $r > 0 \quad 0 < \theta < 2\pi$

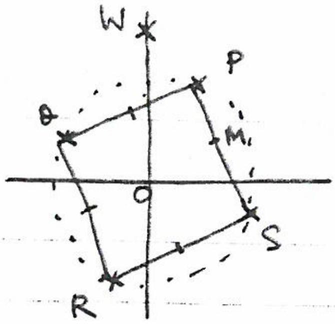
$\theta = \frac{\phi + 2\pi k}{n} = \frac{\frac{3\pi}{2} + 2\pi k}{4} \qquad k = 0, 1, 2, 3$

$\therefore$ Arguments are $\frac{3\pi}{8}, \ \frac{3\pi}{8} + \frac{2\pi}{4}, \ \frac{3\pi}{8} + \frac{4\pi}{4}, \ \frac{3\pi}{8} + \frac{6\pi}{4}$

$= \frac{3\pi}{8}, \ \frac{7\pi}{8}, \ \frac{11\pi}{8}, \ \frac{15\pi}{8} \qquad 0 < \theta < 2\pi$



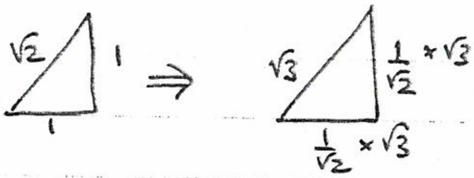
(ii)



M midpoint PS.

$OP = \sqrt{3}$

$\therefore \; OM = \dfrac{\sqrt{3}}{\sqrt{2}}$



$\therefore$ m has modulus $\sqrt{\frac{3}{2}}$

Arg m given by $\frac{1}{2}\left(\frac{3\pi}{8} - \frac{\pi}{8}\right) = \frac{\pi}{8}$ $\quad \therefore \; M = \sqrt{\frac{3}{2}}\, e^{\frac{\pi}{8}j}$

So $w = m^4$ $\quad$ so modulus of $w = |w| = \left(\sqrt{3/2}\right)^4 = \frac{9}{4}$

argument $(w) = 4 \times \frac{\pi}{8} = \frac{\pi}{2}$

$\therefore \; W = \frac{9}{4} e^{\frac{\pi}{2} j} = \frac{9}{4} j$

FP2 January 2011.

2 (a) (i) $z = \cos\theta + j\sin\theta$ $\quad z^n = (\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$

$z^{-n} = (\cos\theta + j\sin\theta)^{-n} = \cos n\theta - j\sin n\theta$

$\therefore\ z^n + z^{-n} = 2\cos n\theta$

$z^n - z^{-n} = 2j\sin n\theta$

(ii) $(z + z^{-1})^6 = (2\cos\theta)^6 = 64\cos^6\theta$

But $(z+z^{-1})^6 = z^6 + 6\frac{z^5}{z} + 15\frac{z^4}{z^2} + 20\frac{z^3}{z^3} + 15\frac{z^2}{z^4} + 6\frac{z}{z^5} + \frac{1}{z^6}$

$\therefore\ 64\cos^6\theta = \left(z^6 + \frac{1}{z^6}\right) + 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) + 20$ ①

$= 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$

$\therefore\ \cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{20}{64}$

$\therefore\ \cos^6\theta = \frac{1}{32}\left(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right)$

(iii) $(z - z^{-1})^6 = (2j\sin\theta)^6 = 64j^6\sin^6\theta = -64\sin^6\theta$

$\therefore\ -64\sin^6\theta = \left(z^6 + \frac{1}{z^6}\right) - 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) - 20$

$= 2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta - 20$ ②

① + ②

$64(\cos^6\theta - \sin^6\theta) = 4\cos 6\theta + 60\cos 2\theta$

$\therefore\ \cos^6\theta - \sin^6\theta = \frac{1}{16}\cos 6\theta + \frac{15}{16}\cos 2\theta$

(b) $w = 8e^{j\pi/3}$ $\quad z_1$ is a square root of w and z_2 cube root of w.

w has modulus $= |w| = 8$; $\arg(w) = \pi/3$ $\quad w = 8e^{\pi j/3}$

z_1 has modulus $= \sqrt{8}$ $\quad \arg(z_1) \Rightarrow (e^{\pi j/3})^{1/2} = e^{\frac{\pi j}{6}}$

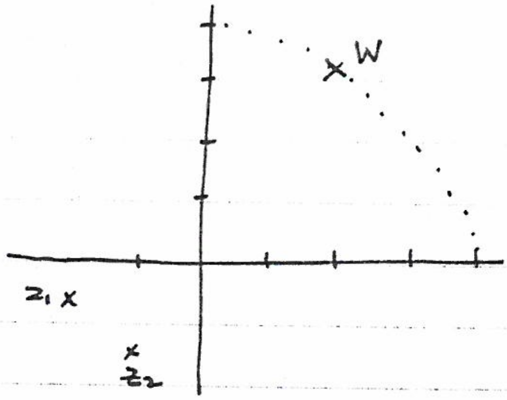
$\arg(z_1)$ could be $\frac{\pi}{6}$ but z_1 in 3rd quadrant so $\arg(z_1) = \frac{\pi}{6} + \pi$

$\therefore\ z_1 = \sqrt{8}\,e^{\frac{7\pi}{6}j}$ (since 2nd of 2 roots is $+\pi$)

z_2 has modulus $= 8^{1/3} = 2$ $\quad \arg(z_2) \Rightarrow (e^{\pi j/3})^{1/3} = e^{\frac{\pi j}{9}}$

$\arg(z_2)$ could be $\frac{\pi}{9}$ but z_2 in 3rd quadrant so $\arg(z_2) = \frac{\pi}{9} + \frac{4\pi}{3}$

$\therefore\ z_2 = 2e^{\frac{13\pi}{9}j}$ (since 2nd of 3 roots is $+\frac{2\pi}{3}$; 3rd is $\frac{4\pi}{3}$)



(ii) product $z_1 z_2$

= multiply moduli

add the arguments

if $z_1 = \sqrt{8}\, e^{\frac{\pi}{6} j}$ $\quad z_2 = 2\, e^{\frac{13\pi}{9} j}$

$\therefore |z_1 z_2| = \sqrt{8} \times 2 = 4\sqrt{2}$

$\arg(z_1 z_2) = \frac{7\pi}{6} + \frac{13\pi}{9} = \frac{47\pi}{18} = \frac{11\pi}{18}$

$\therefore z_1 z_2 = 4\sqrt{2}\, e^{\frac{11\pi}{18} j}$

which lies in 2^{nd} quadrant $(9 < 11 < 18)$

FP2 June 2011.

2 (a) let $z = \cos\theta + i\sin\theta$ $\therefore$ $z^5 = (\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

but $(\cos\theta + i\sin\theta)^5 = c^5 + 5c^4 is + 10c^3(is)^2 + 10c^2(is)^3 + 5c(is)^4 + (i$

$\cos 5\theta + i\sin 5\theta = c^5 + 5ic^4 s - 10c^3s^2 + 10ic^2s^3 + 5cs^4 + is^5$

comparing real parts

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

comparing imag. parts

$$\sin 5\theta = 5\cos^4\theta\sin\theta + 10\cos^2\theta\sin^3\theta + \sin^5\theta$$

if $t = \tan\theta$

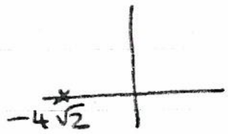
$$\tan 5\theta = \frac{5c^4s + 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4} \qquad \div c^5$$

$$\therefore \tan 5\theta = \frac{5\frac{s}{c} + 10\frac{s^3}{c^3} + \frac{s^5}{c^5}}{\frac{c^5}{c^5} - 10\frac{s^2}{c^2} + 5\frac{s^4}{c^4}}$$

$$\therefore \tan 5\theta = \frac{5t + 10t^3 + t^5}{1 - 10t^2 + 5t^4}$$

$$\therefore \tan 5\theta = \frac{t(t^4 + 10t^2 + 5)}{(5t^4 - 10t^2 + 1)}$$

(b) (i) 5^{th} roots of $-4\sqrt{2}$ $r > 0$ and $0 \le \theta < 2\pi$



let $w = -4\sqrt{2} = -\sqrt{32}$

$r = \sqrt[5]{32} = \sqrt{2}$ $\arg(w) = \pi$

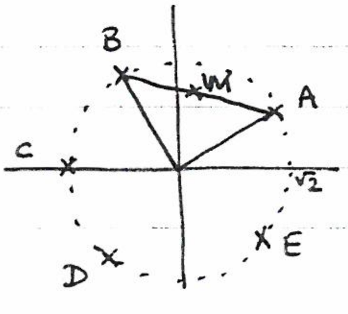
$\therefore \theta = \dfrac{\phi + 2\pi k}{5}$ $k = 0, 1 \ldots 4$ $\therefore \theta = \dfrac{\pi}{5} + \dfrac{2\pi k}{5}$

$\therefore \theta = \dfrac{\pi}{5}, \dfrac{3\pi}{5}, \dfrac{5\pi}{5}, \dfrac{7\pi}{5}, \dfrac{9\pi}{5}$

$\therefore 5^{th}$ roots of w are $\sqrt{2}e^{\pi j/5}, \sqrt{2}e^{3\pi j/5}, \sqrt{2}e^{\pi j}, \sqrt{2}e^{7\pi/5 j}, \sqrt{2}e^{\frac{9\pi}{5}j}$

(ii)

(iii)

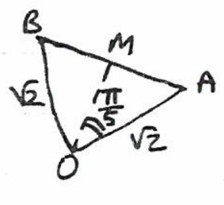


A, B, C, D, E vertices of regular pentagon.

W is midpoint of AB

which has argument $\frac{1}{2}\left(\frac{\pi}{5} + \frac{3\pi}{5}\right) = \frac{2\pi}{5}$

$\hat{AOB} = \frac{1}{2}\left(\frac{2\pi}{5}\right) = \frac{\pi}{5}$



$\frac{OM}{OA} = \cos\left(\frac{\pi}{5}\right)$ so $OM = \sqrt{2}\cos\left(\frac{\pi}{5}\right)$ and $OM = |w|$

$\therefore \quad w = \sqrt{2}\cos\left(\frac{\pi}{5}\right) e^{\frac{2\pi}{5}j}$

(iv) w is n^{th} root of real number a $\quad n > 0$

$\therefore \quad w^n = \left(\sqrt{2}\cos\left(\frac{\pi}{5}\right) e^{\frac{2\pi}{5}j}\right)^n$

To be a real number implies $\left(e^{\frac{2\pi}{5}j}\right)^n = \left(\cos\left(\frac{2\pi}{5}\right) + j\sin\left(\frac{2\pi}{5}\right)\right)^n$

$\Rightarrow \sin\left(\frac{2\pi n}{5}\right) = 0$

$\therefore \quad n = 5 \quad \Rightarrow \sin\left(\frac{2\pi n}{5}\right) = 0$

To find a

$\therefore$ If $n=5 \quad (w^5) = \left(\sqrt{2}\cos\left(\frac{\pi}{5}\right) e^{\frac{2\pi}{5}j}\right)^5$

$w^5 = 2^{5/2}\cos^5\left(\frac{\pi}{5}\right) e^{2\pi j}$

$\therefore \quad w^5 = 2^{5/2}\cos^5\left(\frac{\pi}{5}\right)$ since $e^{2\pi j} = 1$

FP2 January 2012.

2(a) $C = 1 + a\cos\theta + a^2\cos 2\theta + \ldots$

$S = a\sin\theta + a^2\sin 2\theta + a^3\sin 3\theta + \ldots$

$\therefore C + jS = 1 + a(\cos\theta + j\sin\theta) + a^2(\cos 2\theta + j\sin 2\theta) + a^3(\cos 3\theta + j\sin 3\theta) + \ldots$

$= 1 + ae^{j\theta} + a^2e^{j2\theta} + a^3e^{j3\theta} + \ldots$

This is a gp with "a" $= 1$ and "r" $= ae^{j\theta}$

$$S_\infty = \frac{a}{(1-r)} = \frac{1}{(1-ae^{j\theta})} = \frac{(1-ae^{-j\theta})}{(1-ae^{j\theta})(1-ae^{-j\theta})}$$

$$= \frac{1-ae^{-j\theta}}{1-ae^{-j\theta}-ae^{j\theta}+a^2}$$

$$= \frac{1-a(\cos\theta - j\sin\theta)}{1+a^2-a(\cos\theta - j\sin\theta)-a(\cos\theta + j\sin\theta)}$$

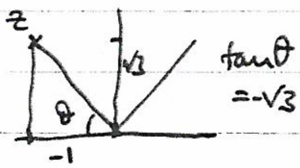
$$= \frac{1-a\cos\theta + aj\sin\theta}{1+a^2-2a\cos\theta}$$

comparing real parts:

$$C = \frac{1-a\cos\theta}{1+a^2-2a\cos\theta} \quad \text{and}$$

comparing imag parts:

$$S = \frac{a\sin\theta}{1+a^2-2a\cos\theta}$$



(b) $z = -1 + j\sqrt{3}$ $\quad |z| = r = \sqrt{1+3} = 2$

$\arg(z) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore z = 2e^{\frac{2\pi j}{3}}$

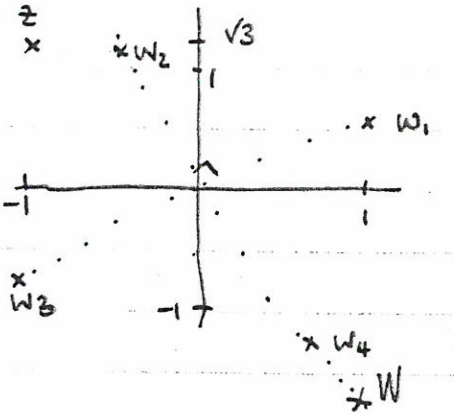
The 4^{th} roots of z $\quad w = z^{1/4} = 2^{1/4}e^{\pi j/6}$

$\therefore \mathrm{mod}(w) = 2^{1/4} \quad \arg(w) = \frac{\pi}{6}$

$$\therefore \theta = \frac{\frac{\pi}{6} + 2\pi k}{4} \quad \text{for } k = 0, 1, 2, 3$$

$\therefore \theta = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

$\therefore$ roots are $2^{1/4}e^{\pi j/6}, 2^{1/4}e^{\frac{2\pi j}{3}}, 2^{1/4}e^{\frac{7\pi j}{6}}, 2^{1/2}e^{\frac{5\pi j}{3}}$



Product of 4^{th} roots

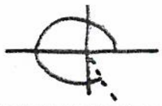
– modulus is product of moduli

– argument is sum of args

$\therefore \quad r = (2^{1/4})^4 = 2$

$\arg(\text{product}) = \frac{\pi}{6} + \frac{2\pi}{3} + \frac{7\pi}{6} + \frac{5\pi}{3}$

$= \frac{11}{3}\pi = 5\frac{\pi}{3}$



$\frac{11\pi}{3} - \frac{6\pi}{3}$

$= \frac{5\pi}{3}$

$\therefore \quad W = 2e^{\frac{5\pi}{3}j}$